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OCD ltr, 27 Oct 1972

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Technical Report

GROUND SHOCK AND THE SURVIVAL OF THE CONTENTS OF PERSONNEL SHELTERS

Resistance of Human and Inanimate Contents of Hardened Shelters to Nuclear—Induced Ground Motion

Prepared for:

OFFICE OF CIVIL DEFENSE
OFFICE OF THE SECRETARY OF THE ARMY
WASHINGTON, D.C. 20301

CONTRACT OCD-PS-64-201
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STANFORD RESEARCH INSTITUTE
MENLO PARK, CALIFORNIA

SRI

STANFORD RESEARCH INSTITUTE

MENLO PARK, CALIFORNIA

May 1968

Addendum to:

Technical Report

GROUND SHOCK AND THE SURVIVAL
OF THE CONTENTS OF
PERSONNEL SHELTERS (Part I)

Prepared for:

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By: J. R. Rempel

SRI Project MU-4949-431

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LIKELY RANGE OF RESPONSE
SPECTRAL ENVELOPES CORRESPONDING TO SYNTHETIC WAVEFORMS

Because of the great number of uncertainties is any estimate of the response of a general geologic environment to nuclear explosion, a response spectrum computed for a given locality and for given conditions of burst should show a range of likely variation of each ordinate if it is to be of optimum use. In this supplement such ranges are indicated for the spectra reported in Section VE of the SRI report, "Ground Shock and the Survival of the Contents of Personnel Shelters," November 1967, prepared for the Office of Civil Defense (OCD). These spectra are regarded as the best estimates available from existing knowledge of the response in five specific areas within the United States to nuclear bursts at given points in the areas.

There are three sources of uncertainty: (1) lack of understanding of the phenomena of transmission of complex waveforms through a complex geology, (2) lack of detailed knowledge of the geology over any large area, and (3) lack of knowledge of pertinent mechanical characteristics of any given soil. We have not attempted at this time to allow for variations introduced from the first source; there are in existence unambiguous empirical rules for determining ground response that have been extracted from weapons test experience, and we have adhered to these rules. What we have done is try to determine the effect on computed spectra of certain reasonable variations in geology and material properties. Fortunately, because of decoupling of influences, this is a fairly straightforward thing to do.

According to the empirical rules, ground motion is the sum of two parts or waves: airslap and ground transmitted. The airslap shows only the influence of the blast overpressure and the soil at the site. Peak downward ground speed is inversely related to soil "shock impedance." In the accompanying spectra we have allowed a ± 25 percent variation in

this parameter. (The characteristics of the airblast itself are, of course, well established and no variation in them was considered.) This large range was chosen because "shock impedance" is a quantity that must be inferred from acoustic properties, and the connection between shock and acoustic or seismic behavior is quantitatively known for only a very few soils.

The second component, the ground-transmitted wave, is influenced by all the soil between the site and the burst, as well as the soil at the site. In the first place, the presence or absence of the ground-transmitted wave at a site and its time of arrival with respect to the air blast depend upon shock speeds at every depth between the burst and the site. Second, the ground wave is undulatory and its time characteristics or "period" also depend strongly on the whole geology between the burst and site. However, if the ground wave does appear at a given site, its amplitude is fairly clearly linked to overpressure and surface soil conditions at the site. Thus, we have not only allowed peak ground speed in the undulating component at each site to vary ± 25 percent, but we have had to consider a large variation in period and relative time of arrival; at some sites we have added or subtracted the ground-transmitted wave altogether. Generally extreme changes in timing have been computed using a ± 25 percent variation in the seismic speed in the geologic layer responsible for the transmission of the ground wave.

Ignoring unimportant fine structure, superposition of the two components has generally led either to simple superposition of two spectral envelopes, one corresponding to airslap and the other corresponding to the ground wave, or to destructive interference, i.e., reduction of spectral ordinates in a narrow range due to the opposition of an upward ground-transmitted motion and a downward airslap motion. There are, of course, downward components in the rolling motion, and, in some of the variations considered, constructive interference has occurred but this phenomenon is not so prominent as destructive interference.

The high frequency limits of airslap spectral envelopes depend upon the value taken for the peak acceleration in the initial downward motion.

Following the rules, we have made this inversely proportional to seismic speed; however, this is an extremely doubtful procedure because of the importance to acceleration of inelastic loss mechanisms in most soils. The high frequency bounds on our spectral envelopes are not well established and should not be taken without reservation.

In the accompanying graphs the heavy line traces the most likely spectral envelope and is based on the same parameter values as the spectra shown in Section VE of the report mentioned above; the shaded areas on each side show the likely variation as computed according to the foregoing principles. These areas enclose a number of spectra computed for each overpressure and city. The light lines at a slope of + 1 are coordinates corresponding to maximum relative oscillator displacement, x_M; the lines at a slope - 1 correspond to loci of constant maximum acceleration $\omega^2 x_M$. Some spectral ordinates shown by the heavy line in this supplement are slightly higher than corresponding ordinates presented in the SRI report prepared for OCD because a computer with a larger word size was used in their calculation than was available during the preparation of the report.

Figure 1

VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM

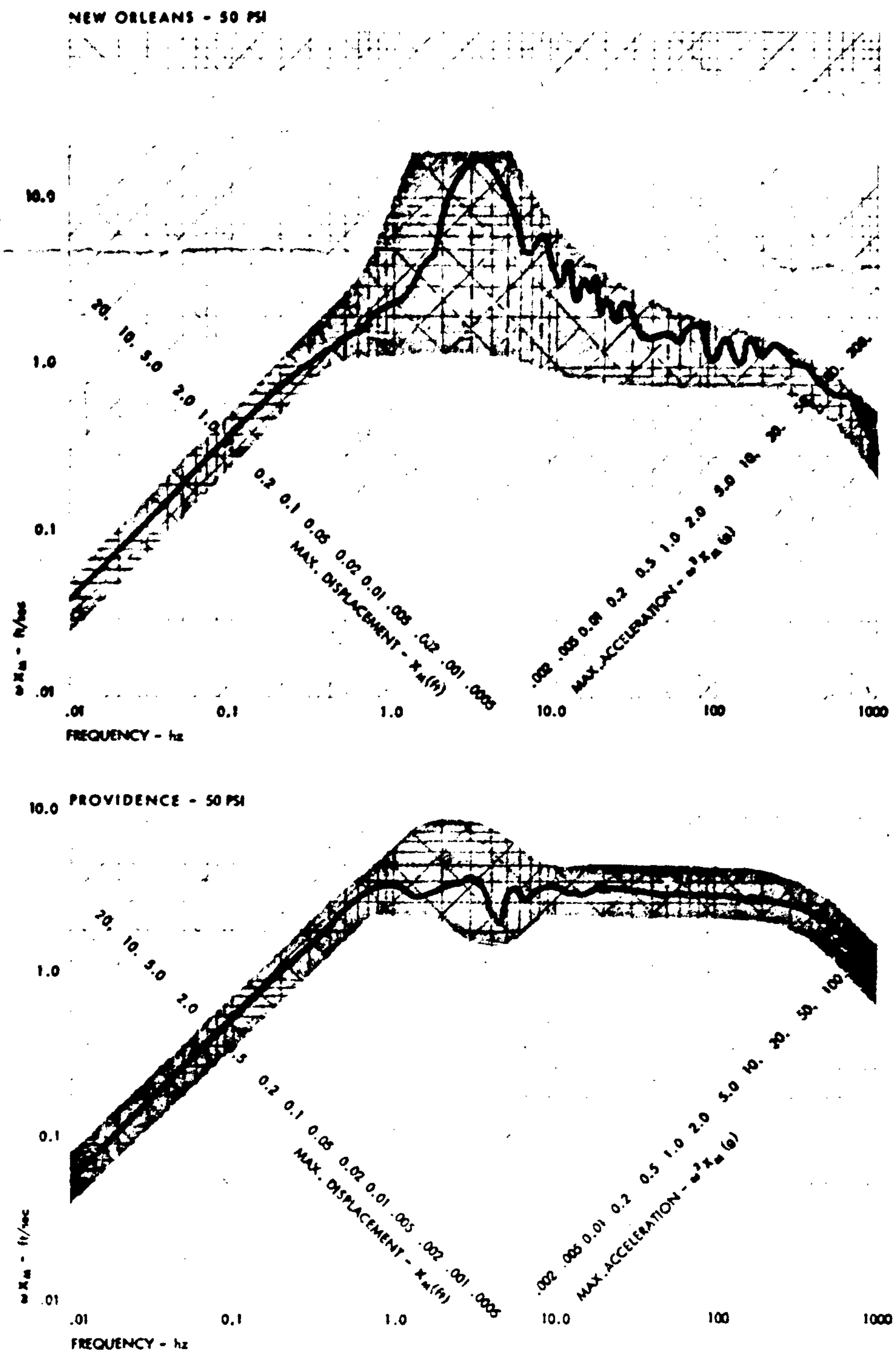
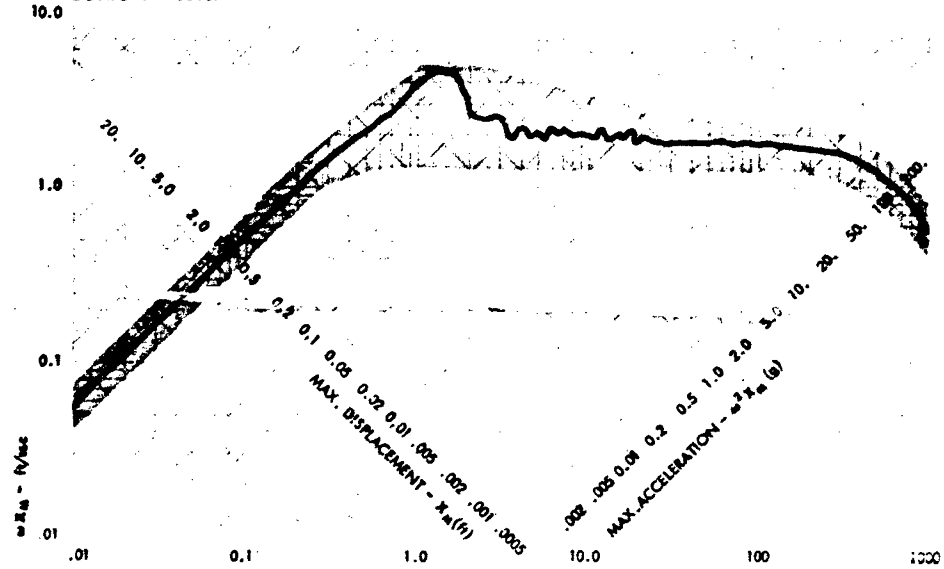


Figure 2

VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM

DETROIT - 50 PSI



NEW ORLEANS - 25 PSI

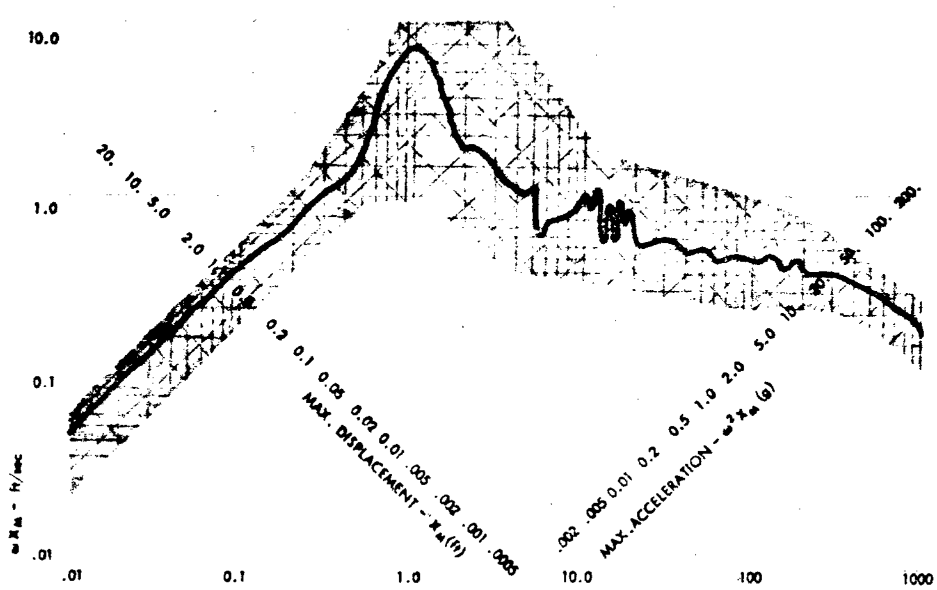


Figure 3

VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM

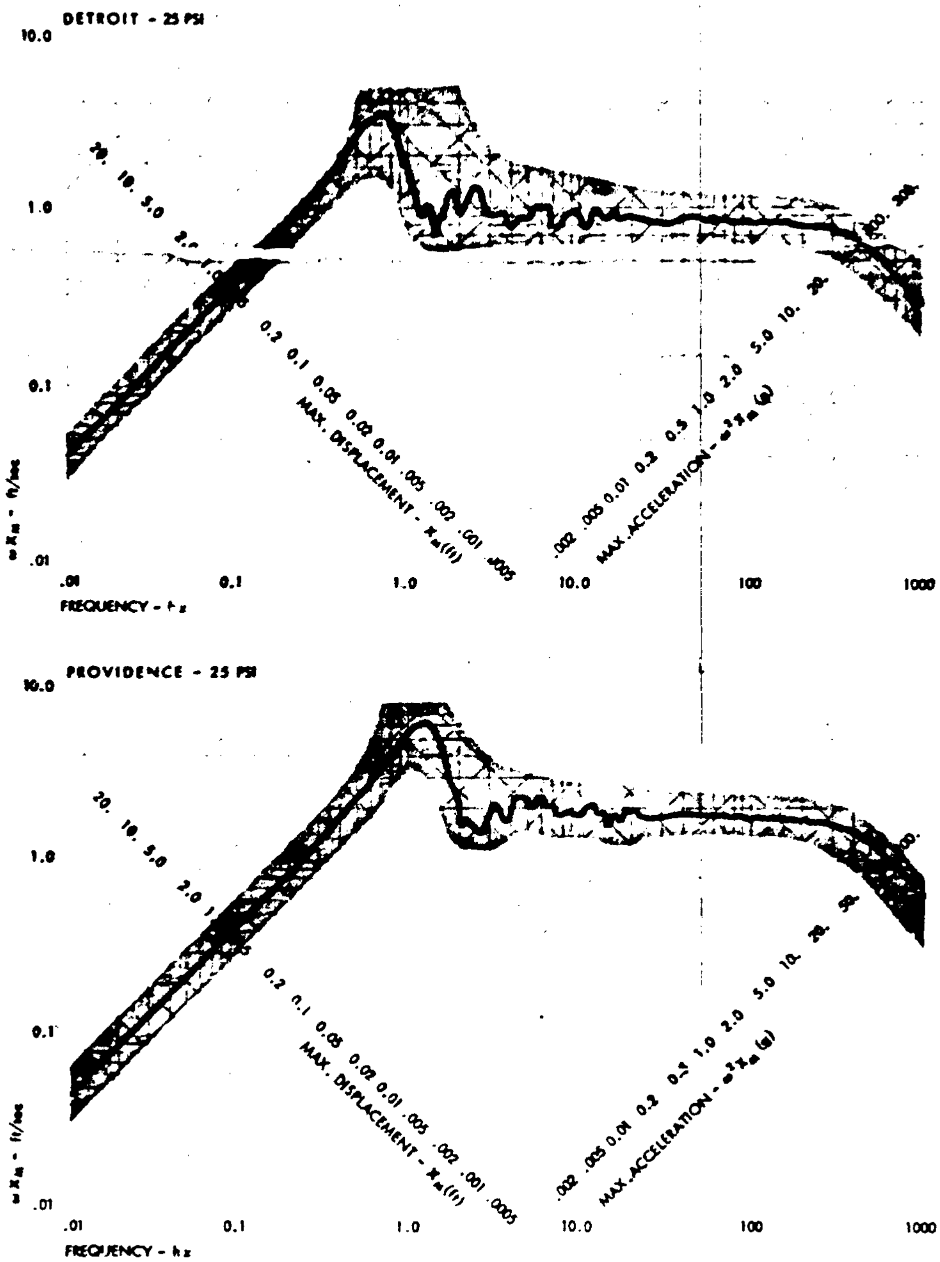


Figure 4

VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM

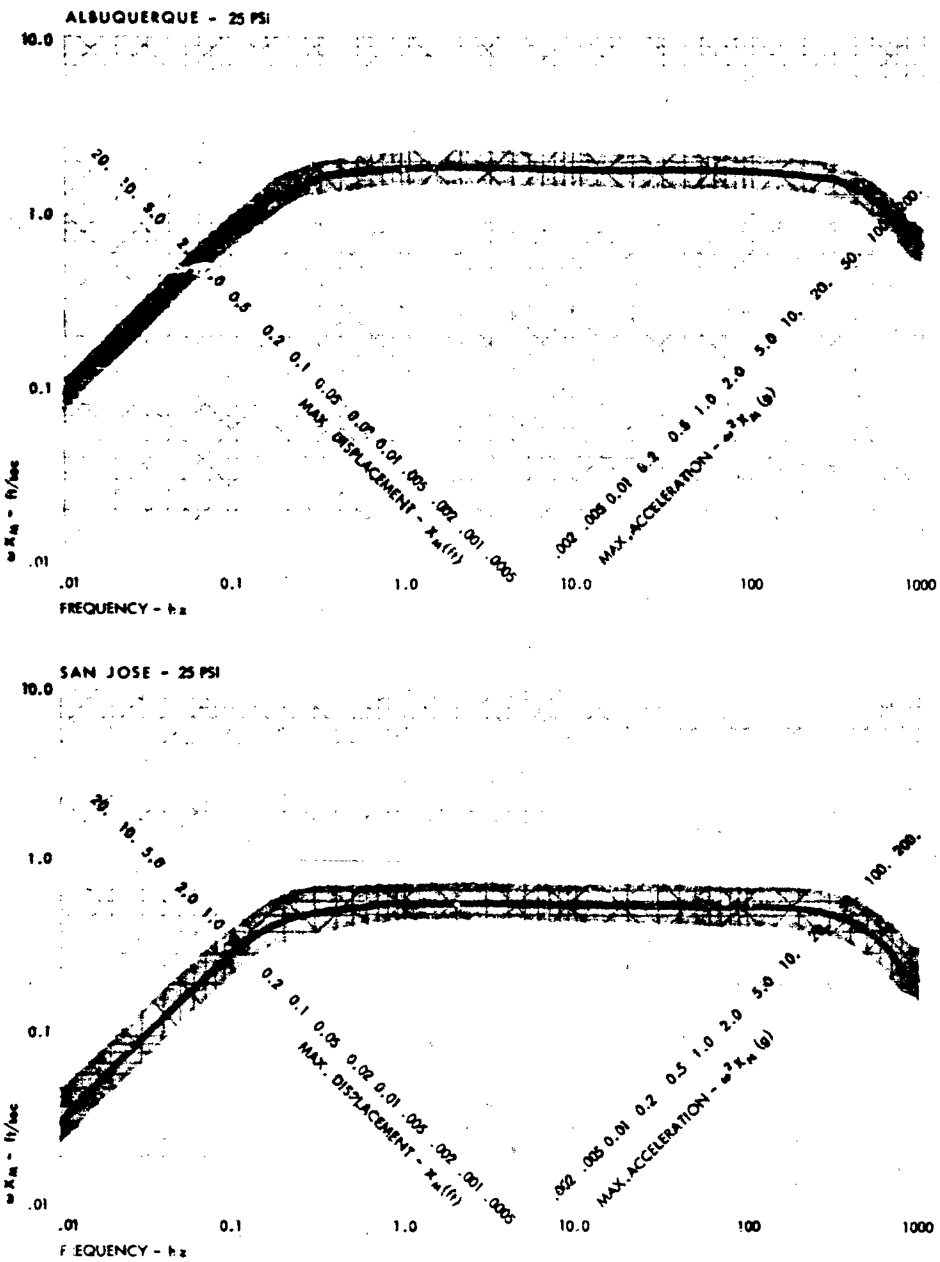
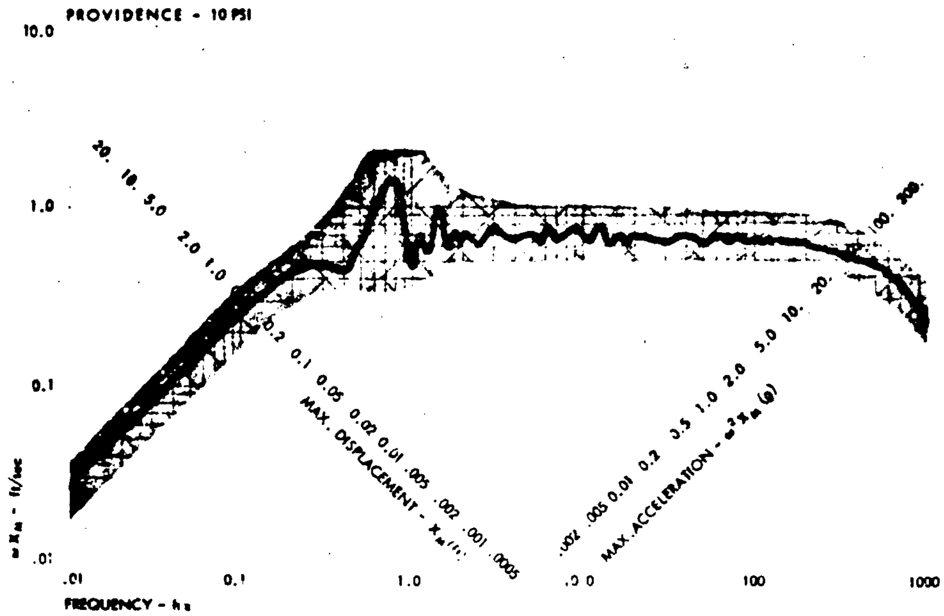


Figure 5

VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM



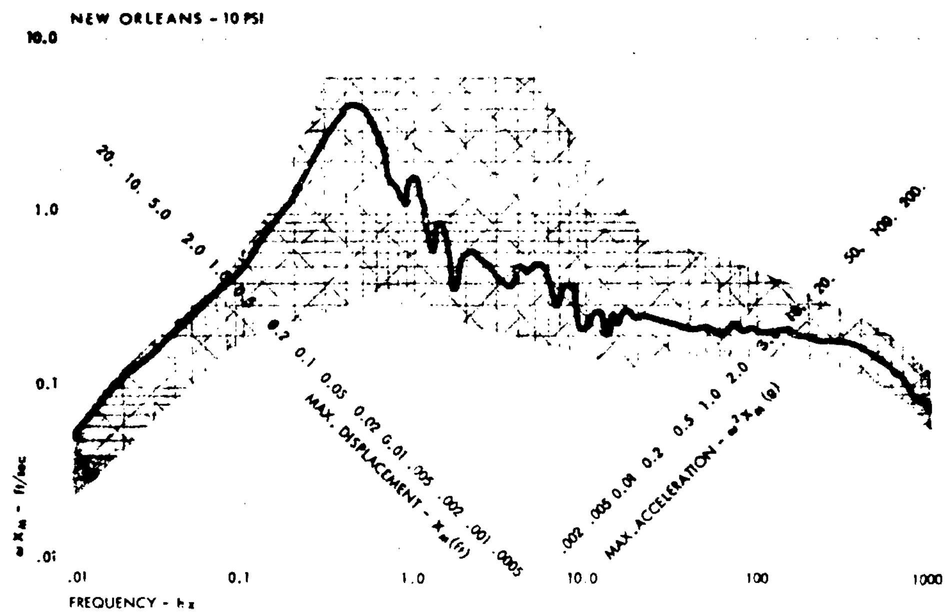
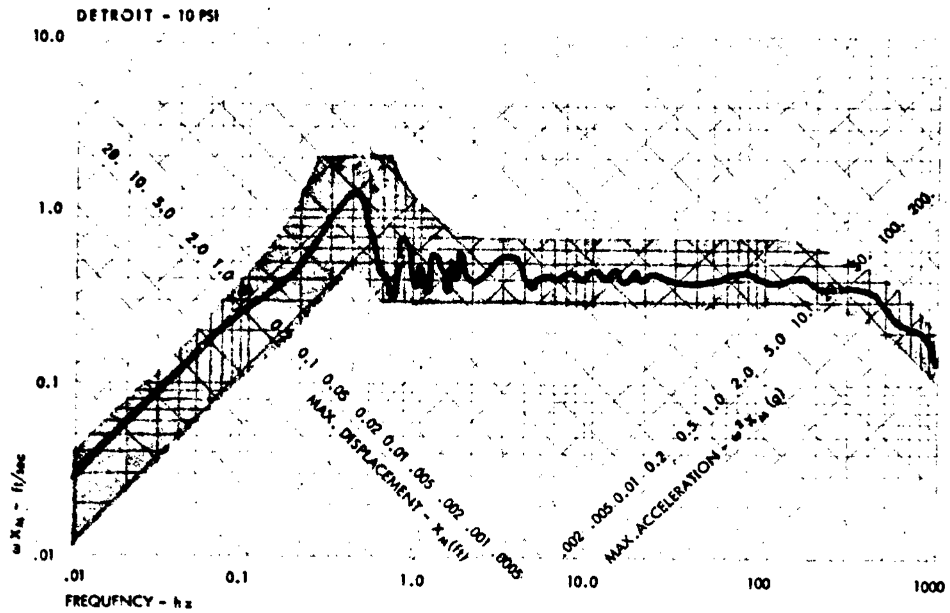


Figure 6

VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM



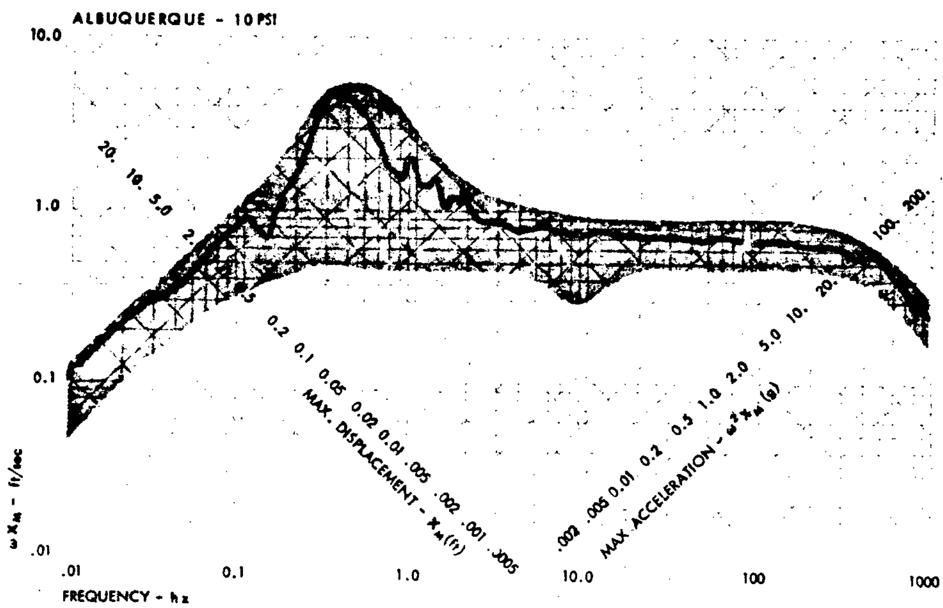
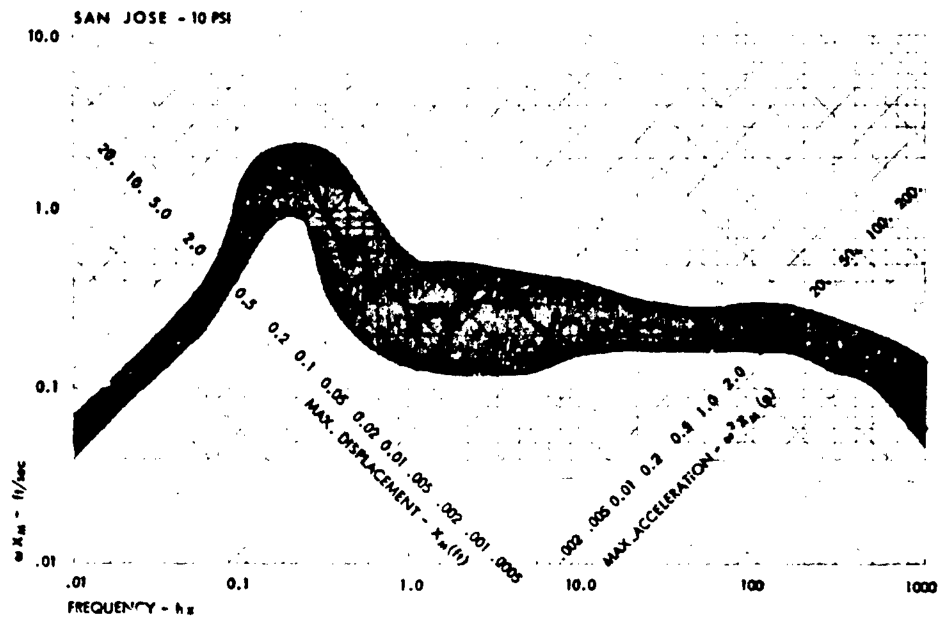


Figure 7

VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM



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November 1967

Technical Report

GROUND SHOCK AND THE SURVIVAL OF THE CONTENTS OF PERSONNEL SHELTERS

Resistance of Human and Inanimate Contents of Hardened Shelters to Nuclear—Induced Ground Motion

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By: J. R. REMPEL

SRI Project MU-4949-431

This report has been reviewed in the Office of Civil Defense and approved for publication. Approval does not signify that the contents necessarily reflect the views and policies of the Office of Civil Defense.

Approved: MARJORIE W. EVANS, DIRECTOR
POULTER LABORATORY FOR HIGH PRESSURE RESEARCH

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GROUND SHOCK AND THE SURVIVAL OF THE CONTENTS OF PERSONNEL SHELTERS

Resistance of Human and Inanimate Contents of Hardened Shelters to Nuclear-Induced Ground Motion

DETACHABLE ABSTRACT

No rules are readily available for estimating the potential damage to the human body caused by nuclear-induced ground motion. Bomb shelter experimenters in World War II were not faced with the problem under the energy inputs of chemical explosives. With the advent of nuclear weapons and their considerable greater energy input to the earth, ground shock became an important factor in the survival of humans and equipment in an underground shelter under nuclear attack. This report is an attempt to summarize the hazard created by ground motion stemming from a nearby nuclear explosion to people and equipment inside a hardened personnel or civil defense command and control shelter which itself withstands both the airblast and ground shock environment. Specifically, this study attempts to determine from available literature at which amplitude and in what frequency range ground shock will be likely to produce disabling injuries to the human body and damage to equipment.

No person or pertinent equipment has been exposed within an underground structure to strong ground motion from a nuclear burst for the purpose of exploring the motion hazard. Results of animal experiments in such an environment are obscured by other than ground motion effects and do not apply to people directly. For these reasons, appraisal of the strong ground motion threat to people and equipment must generally be based on extrapolation from experiments in other environments. To guide this extrapolation, elementary use of two theoretical models was made: the simple harmonic oscillator and the steady-state, one-dimensional shock wave.

Since there are practical economic limits on shelter construction, it was assumed for this study that the shelter is not deeply buried but that it is at least mainly underground and is located at a 50-psi range from the nuclear detonation. It is also assumed that the shelter not only remains intact during the attack but that it protects its contents from increases in air pressure. It is not, however, considered to be (as a whole) shock or vibration isolated; it both transmits ground shock to its contents and responds to the shock motion. It is further assumed that the people inside do not have time to adopt protective postures and that no heavy objects that could act as missiles are on the shelter walls.

Human response to one-sided acceleration pulses applied to the whole body has been sufficiently studied to produce a pair of values, A_o, the limiting value for the average acceleration, and V_o, the limiting value for total speed change, that must be exceeded for probable injury. Observations of human response to impacts of duration within the range 10 msec to 1.0 sec indicate values of A_o between 7 and 20 g and values of V_o between 10 and 80 ft/sec, depending upon bodily orientation.

Calculating from these criteria we deduce that it will be possible to shelter a general population at 50 psi and greater ranges although some injury will occur, stemming chiefly from toppling (i.e., loss of balance). There will be other mechanisms of injury to consider at ranges between 50 and 200 psi in alluvial or composite soils. Toppling remains the most likely threat to safety up to roughly 4000 psi in hard rock. (Protective postures--bent knees, prone forms, or resting on hands and knees--will offer greater resistance to damage-inducing motions in all media than will standing.)

To minimize as much as possible bodily injury due to toppling, low impedance covering should be installed on shelter walls and floors, and sharp corners and points should be eliminated within the inhabited space.

Equipment response cannot be summarized so succinctly as the response of humans. However, small pieces of equipment (pumps, fans, motors, etc.) appear, under tests specified by the DOD, to be able to survive overpressures up to 50 psi. Large pieces of equipment, on the other hand, may be susceptible to damage below a frequency between 10 and 50 Hz, depending on overpressure. Hard mounting of all equipment in shelters is recommended, as well as the continued and more widespread use of tests to determine equipment resistance to motion. Wider use of drop or variable duration tests should be made to simulate ground motion more realistically than can hammer blows or underwater explosions.

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ILLUSTRATIONS (Continued)

ILLUSTRATIONS (Concluded)

SYMBOLS

A	ground acceleration
A_M	peak ground acceleration
A_o	highest tolerable value of average acceleration applied to the base of an oscillator or to the human body (gravities)
D	ground displacement (ft)
I	shock impedance
K	Dieckmann's index of tolerance to steady vibration
P	pressure
P_M	peak pressure
P_o	ambient pressure
R	distance along the surface from ground-zero or distance from explosion or ratio
T	duration of acceleration applied to base of harmonic system
T_n	natural period of an oscillator or equivalent oscillator (sec)
T_o	"corner" frequency of a sensitivity diagram, $T_o = V_o/A_o$
T_1	duration of positive airblast overpressure
T_2	characteristic time of ground-transmitted motion (msec or sec)
U	shock wave speed
V	ground speed (ft/sec)
V_M	maximum ground speed (ft/sec)
V_o	highest tolerable value of speed change applied to the base of an oscillator or to human body (ft/sec)
W	weapon yield or weight of explosive

SYMBOLS (Continued)

a	average acceleration of base of a harmonic system
c	sound or elastic wave speed
d	layer thickness
f	frequency (Hz) or fraction
f_n	natural frequency of an oscillator or equivalent oscillator (Hz)
f_1	natural frequency of primary oscillator when uncoupled
f_2	natural frequency of secondary oscillator when uncoupled
k_1,k_2	spring constants in primary and secondary subsystems of coupled two-degree-of-freedom harmonic oscillator
m_1	mass in primary subsystem
m_2	mass in secondary subsystem
n	ratio of tolerable distortion in primary to tolerable distortion of secondary of a two-degree-of-freedom coupled system
n	integer, or, as subscript, "natural"
r	ratio of primary to secondary frequencies f_1/f_2
s	displacement
t	time (sec)
u	particle or material speed
u_f	"fracture" speed or least relative speed between body component and heavy object needed to break component
v	speed change in base of a harmonic system
x_M	maximum displacement of oscillator mass with respect to base from normal position ("distortion" or "strain")
z	mechanical impedance
α	amplification of spectral ordinate
β	fraction critical damping in secondary system

SYMBOLS (Concluded)

ΔGR	difference in ground range between observing station and location where seismic motion overtakes airblast (ft)
ν	fraction of critical damping
ρ	density
σ	peak stress
σ_t	tensile strength
θ	a time constant, angle
ω	circular frequency (sec^{-1})

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The author is particularly grateful to those who have willingly taken their own time to help educate him in the realities of shock testing and the emergency operating center: R. W. Crawford, John W. Foss, R. W. Mayo, and Clinton Shafer, Jr., of the Bell Telephone Laboratories, Whippany, N.J.; Hal B. Hansen, Bell Telephone System, New York City; C. L. Wickstrom, Pacific Telephone Company, San Francisco; C. D. Morrissey and James Noonan, Prager-Kavanagh-Waterbury, New York City; Paul Sprehe, Consulting Engineer, Oklahoma City; John Canada, Deputy Director for Civil Defense, State of Oklahoma; Donald N. Coupard, Consulting Engineer, Rockville, Md.; Dr. Sal Giannoccolo, K. W. Rosenthal, and Carl Shrader, West Coast Shock Test Facility, San Francisco Naval Shipyard.

The author's colleagues, Robert Kiessling, George M. Muller and Fred M. Sauer also supplied valuable guidance. John H. Hannigan's knowledge of naval machinery was essential.

RESISTANCE OF HUMAN AND INANIMATE CONTENTS OF HARDENED SHELTERS TO NUCLEAR-INDUCED GROUND MOTION

I INTRODUCTION

The question of protection of the civil population in the event of nuclear attack raises a multitude of questions ranging in scope from political to economic. One of the questions in the latter category is "What hazards are presented by the ground motion stemming from a nearby nuclear explosion to people and equipment inside a personnel or civil defense command and control shelter which itself withstands both the airblast and ground shock environment?" The question is an economic one because people and equipment properly shock isolated and/or far enough away from a small enough nuclear detonation would be in no danger at all, and it also has deep sociological implications since the overall hazard to the civil population must be viewed not only in terms of the hazard to a single individual but in terms of the degradation of the social structure of the sheltered community which may occur as the number of casualties increases.

The answer to this and similar civil defense questions depend a great deal on the structure of the postulated attack, i.e., the number, yield, and height of burst of the attacking nuclear warheads which successfully penetrate our active defense systems and their distribution in time and space in relation to the location of personnel shelters; and on the preparedness of the civil population in expecting imminent attack. By itself, the question of the hazard presented by ground shock stands puny indeed against the goliath of uncertainty in the attack parameters; the true significance of a quantitative answer to the ground shock question can only be found in an integrated assessment of damage which

includes as a major element of the study a variation in attack parameters and uses actual sitings and geological conditions. Such a study presumes a model, which is at best no more certain than the product of the certainties in its basic elements which are hopefully based on knowledge of the physical principles involved.

By now it should be becoming clear that we have not set ourselves the task of answering the question of how important it is to civil defense to consider ground shock as a damage mechanism; this answer must come from further study beyond this research. Our objective here is to develop an understanding of one of the elements required for further study.

We have focused our primary attention on the quantitative evaluation of the hazard to people and equipment from ground shock, i.e., at what amplitude and in what frequency range will ground shock be likely to produce disabling injuries to the human body and damage to equipment of the type found in Civil Defense command and control centers? This we believe to be currently a more important question than the uncertainty in prediction of ground motion itself since empirically developed rules can be found which, although challengeable, are widely accepted (see for example, Refs. 1 and 2) and which are probably uncertain by less than a factor of 10.

In contrast, no such rules are readily available for damage by ground shock to the human body. Bomb shelter experimenters in World War II were not faced with the problem under the energy inputs of conventional chemical explosives. Rather, of concern to these investigators were mechanical injury to humans due to shelter collapse, carbon monoxide poisoning and heat from surrounding fire, and asphyxiation resulting from dust spallation off the walls (Ref. 3).* With the advent of nuclear

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* It was concluded, incidentally, in Ref. 3, that dust was a hazard only in buildings with masonry or plastered walls; reinforced concrete does not produce enough dust to be harmful. This conclusion regarding concrete has been confirmed in underground shelters during weapons test at Nevada Test Site (Ref. 4).

weapons and their considerable energy input to the earth, ground shock has become an important factor in the survival of humans and equipment in an underground shelter under nuclear attack. This report is an attempt to summarize the response of the human body and certain types of equipment to the environment likely to be created within a hardened underground shelter by ground motion from a nuclear burst. In another portion of the research* we have looked at the feasibility of using ray theory to provide a rapid means of prediction of ground motion where the energy arrives via a refracted path.

No person or pertinent equipment has been exposed within an underground structure to strong ground motion from a nuclear burst for the purpose of exploring the motion hazard. Hardened shelters containing unisolated relay racks, electronic apparatus, some air conditioning equipment, and motor generator sets were used in the neighborhood of the 50 psi range at nuclear weapons tests both in Nevada and Eniwetok and, although no detailed inspection of the contents was made after the shots, no equipment was replaced specifically because of shock or motion damage, so far as known. However, detailed data are lacking. Results of animal experiments in such an environment are obscured by other than ground motion effects and do not apply to people directly. For these reasons, appraisal of the strong ground motion threat to people and equipment must generally be based on extrapolation from experiments in other environments. To guide this extrapolation, elementary use of two theoretical models will be made: the simple harmonic oscillator and the steady-state, one-dimensional shock wave.

In particular, we will use shock response spectra of elastic and visco-elastic systems (Ref. 5) and the empirically-based theory of their meaning in motion simulation (Refs. 6 and 7). Great reliance will be placed on Kornhauser's theory of the tolerance of elastic and near-elastic systems to motion of their bases (Refs. 8 and 9) and to

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* Application of Generalized Ray Analysis to Prediction of Ground Motion from Nuclear Bursts, by K. N. Sawyers and F. M. Chilton, Stanford Research Institute, November 1967.

the underlying idea of equivalent strain in a purely elastic analog of a real system. The theory can be applied to coupled as well as to independent subsystems. Composite sensitivity curves for various two-degree-of-freedom systems responding to rectangular acceleration pulses are given in this report. In rare cases, data are refined enough to allow use of mechanical impedance concepts (Ref. 10). With these theoretical tools, the response to specified ground motion of the human body and of mechanical equipment--idealized as collections of one-degree-of-freedom, harmonic subsystems--is deduced from observed responses to other motions which are easily produced in the laboratory or which have been produced in accidents. Some consideration will be given to the effect of coupling between two component subsystems, but the observational data at this time do not warrant much quantitative thought on nonlinear motion or plastic flow nor on couplings of more than two single degree-of-freedom subsystems. The one exception is the human head; sophisticated theoretical study of the response of the head to blows delivered by various objects is justified but merits a separate study.

Reference will be made to the quasi-static strength of human tissues, and a connection will be drawn between these data and behavior of body components under shock loading.

Specific shelter sites will not be considered but an attempt will be made to appraise the greatest threat likely anywhere under the following assumptions:

Since there are practical economic limits on shelter construction, it is assumed for this study that the shelter is not deeply buried but at least mainly underground and that it is located at a range from the nuclear detonation such that peak airblast overpressure from the detonation is not more than 50 psi. It is also assumed that the shelter itself not only remains intact during the attack but protects its contents from increases in air pressure. It is not, however, considered to be (as a whole) shock or vibration isolated; it will both transmit the ground shock to its contents and respond to the shock motion with

a secondary motion of its own. It is further assumed that the people inside do not have enough forewarning to adopt special attitudes or postures and that common sense precautions have been taken to keep heavy bookcases, pictures, mirrors, or other lightly-fastened or breakable fixtures off the shelter walls.

II CONCLUSIONS

Human response to one-sided acceleration pulses applied to the whole body has been well enough explored to permit definition of a tolerable limit of "equivalent elastic strain" for various bodily orientations, and from this quantity tolerance to all likely kinds of whole body motion can be calculated.

Convenient human tolerance criteria exist in the form of a pair of values, one giving a limiting value for the average acceleration A_o and another for the limiting value for total speed change V_o present in a one-sided acceleration pulse; both values must be exceeded for injury exposure to take place as a result of such a pulse. There may be a different pair of values for each bodily orientation with respect to the imposed motion. There is a weak, theoretical dependence on pulse shape but ordinarily the effect is lost in the presence of experimental uncertainty in the definition of "tolerance limit."

Observations of human response to impacts of duration within the range 10 msec to 1.0 sec indicate values of A_o between 7 and 20 g and values of V_o between 10 and 80 ft/sec, depending on bodily orientation and posture.

There remains some doubt concerning the hazards offered by all the possible kinds of local impact, but generally data can be found to reveal the magnitude of the threat arising from many practical situations, such as, for example, that stemming from a head falling into a hard wall.

Knowledge of the response of equipment to motion can not be summarized so succinctly as the response of humans, but enough testing and observation have been reported to allow certain useful conclusions to be made. Standard specifications for tests adequate for most equipment destined for exposure at overpressures up to 50 psi have been established by the Department of Defense. Many examples of small shelter items have demonstrated survival capacity during these tests, such as, pumps, fans,

motors, and transformers. This does not imply that all examples of these kinds of machinery will have the necessary resistance, and testing of all shelter equipment should be routine. Minor modifications to the equipment are often necessary as the testing proceeds to higher levels of impact. Neither large weight nor large size is any bar in itself to use of the standard tests; a range of test gear exists with a capability of handling test subjects weighing up to 30,000 lbs and occupying a volume 16 x 28 x 9 ft. A great deal of equipment has already been tested by the standard means and many manufacturers hold certification of the results, although access to this evidence may not be casually granted.

However, below a frequency somewhere between 10 and 50 Hz, depending on overpressure, most of the tests that have been applied cease to be adequate for alluvial soils subjected to overpressures above 150 psi and, conceivably, for some composite environments at overpressures at or above 50 psi. Large equipments may be sensitive in this frequency range. This is also the frequency range where interactions, if any, between elements of the shelter structure and the equipment mounted on them will be important. The best way of reaching into the low frequency range is by means of drop tests with sand, plastic, or lead arrestors. Although standard tests of this kind exist, they do not have the capacity of mounting test subjects weighing over 1200 lbs or occupying a volume greater than about 4 ft cube. Improvisation, however, would appear to be relatively simple. Diesel motor-generators, refrigeration and airconditioning equipment, large fans, blowers and batteries, and large, light cabinetry holding relatively heavy relays or transformers would all seem to be susceptible in the low frequency range. The threat at these frequencies is particularly severe in composite environments where the ground-transmitted component can be especially strong.

An added threat arises when shock isolation from airslap is not carried out with proper regard to very strong low-frequency elements in the ground-transmitted wave, which may cause unexpectedly large deflection in the isolating device.

Since application of equipment test results to the present problem rests upon the theory of the response spectral envelope, all the doubt associated with this theory may be transferred to these conclusions. The detailed, quantitative nature of nuclear-induced ground motion has been sufficiently revealed through experiment and theory to forecast within a factor of two the motion likely to be imposed upon underground shelters in most uniform geologic environments by air or surface nuclear bursts. However the response of certain composite environments, particularly those consisting of soft ground media 50 to 200 ft thick underlain by hard rock, can only be estimated heuristically by extrapolation of observations. It turns out that such extrapolation suggests the existence of special conditions for the existence of a greater threat than can be found in "proven" or hitherto observed ground response to nuclear explosions. A great deal of guidance as to the likelihood of these conditions in a given location can probably be obtained by a study of the geologic environment, but no sure forecast can be made with existing knowledge. The degree of certainty increases as the geologic characteristics approach those of either the Nevada Test Site or the Eniwetok Proving Ground. Also the possibility of the importance of Rayleigh waves in uniform hard media and of focussing in stratified environments has been suggested by some writers but not investigated quantitatively.

Qualified by the foregoing comments, this study furnishes the following conclusions with reference to the hazard created by ground motion from a nearby nuclear explosion:

Hazard to Humans

Ground motion will not forbid sheltering a general population at the 50-psi range or at larger ranges, although some injury will generally occur. Since most of this injury will arise from toppling or loss of balance, low impedance covering should be installed over shelter floors and walls, and sharp corners and points eliminated within the inhabited space. Without either personal or structural countermeasures, toppling may in principle lead to quite serious and incapacitating head injury. The extent to which adequate personal countermeasures are a normal

response to strong ground motion has not been studied. Between 50 and 200 psi the possibility of injury through mechanisms other than toppling gradually enters when the soil is of alluvial or composite type. Toppling remains the only very likely threat in hard rock up to roughly 4000 psi. (No consideration has been given to possible focussing or to the Rayleigh wave).

It is speculated that certain composite geologic environments may conceivably offer a direct impact hazard to the human body at distances from the burst less than those corresponding to an overpressure of 20 psi range without provoking a secondary hazard such as toppling. Such hazard has not been demonstrated at Eniwetok or the Nevada Test Site but appears only after considerable extrapolation of observations there.

Standing individuals who bend their knees or lie down, or seated people who distribute their weight to hands and feet will be able to resist motions at even higher overpressure in all environments. There are some data suggesting that the margin of safety for people age 50 or more may be reduced. However, beyond the obvious fact that toppling falls are both more likely to occur to the elderly and more likely to injure them, little quantitative mechanical data have been found for the age dependence of tolerance levels.

It should be emphasized that above the foregoing thresholds injury is not certain but merely possible and above these thresholds some thought should be given by planners to the existence of injury among the sheltered population.

Hazard to Equipment

Hard mounting to all equipment in shelters is a reasonable and probably desirable goal. Achievement will require considerable minor redesign of the equipment itself and both continued and more widespread use of known testing procedures. The advantages of hard mounting are lessened cost of installation and freedom from worry about the reaction of isolators tuned to frequencies which are strongly represented in ground-transmitted, airblast-induced motion.

Completely adequate tests (in the sense of spectral envelopment) are available or can be improvised for use in conjunction with a future program of developing rugged equipment for shelters. The tests to which the Department of the Navy, for example, now regularly subjects shipboard equipment are quite adequate for judging the effects upon small equipment of airslap-induced ground motion up to 400 psi in alluvium and composite soils and up to at least 100 psi in hard rock.

Although equipmental test procedures used in the past are effective for most equipment in most environments up to at least 50 psi, wider use should be made of drop or variable duration methods because they can provide a more realistic simulation of the pertinent ground motion than can hammer blows or even underwater explosions.

III SUMMARY

A. Human Acceleration Data

Nearly all experiments involving controlled acceleration of humans or quantitative studies of accidental acceleration are pertinent to the effect of ground motion on humans and for the research reported here data have been collected ranging from long-lasting (1 to 3 sec duration) constant acceleration in centrifuges to very brief impacts resulting from falls or drops (10 to 20 msec duration). In a few cases these data establish injury thresholds but more often they relate to voluntary acceptance of pain. These observational data will be summarized briefly in terms of the coordinates of a sensitivity diagram: average accelerations (a) as abscissas, and speed change or impact speed (v) as ordinates. Since scales of both coordinates are logarithmic, loci of constant duration are straight lines of slope +1.

Figures S-1 and S-2 have been prepared in this form. Points corresponding to the conditions of observed or inferred impacts are collected in Fig. S-1 and conclusions drawn from this data are represented in Fig. S-2 by "safe" and "unsafe" regions. When both speed change (ordinate) and average acceleration (abscissa) are higher than certain limits, the impact is unsafe; otherwise it is safe. For both theoretical and observational reasons these limits, particularly the average acceleration, can not be given as single values and are shown sometimes as regions in Figs. S-1 and S-2.

The limits depend on bodily orientation with respect to the acceleration vector and the various bounds drawn in Figs. S-1 and S-2 are labelled to indicate the mode of impact. "Longitudinal" means acceleration is applied to the whole body parallel to the spine; "transverse" indicates the vector is normal to the backbone. "Whiplash" is a motion of the head with respect to the shoulders either fore-and-aft or sidewise; the limits shown in the figures for whiplash are not well established.

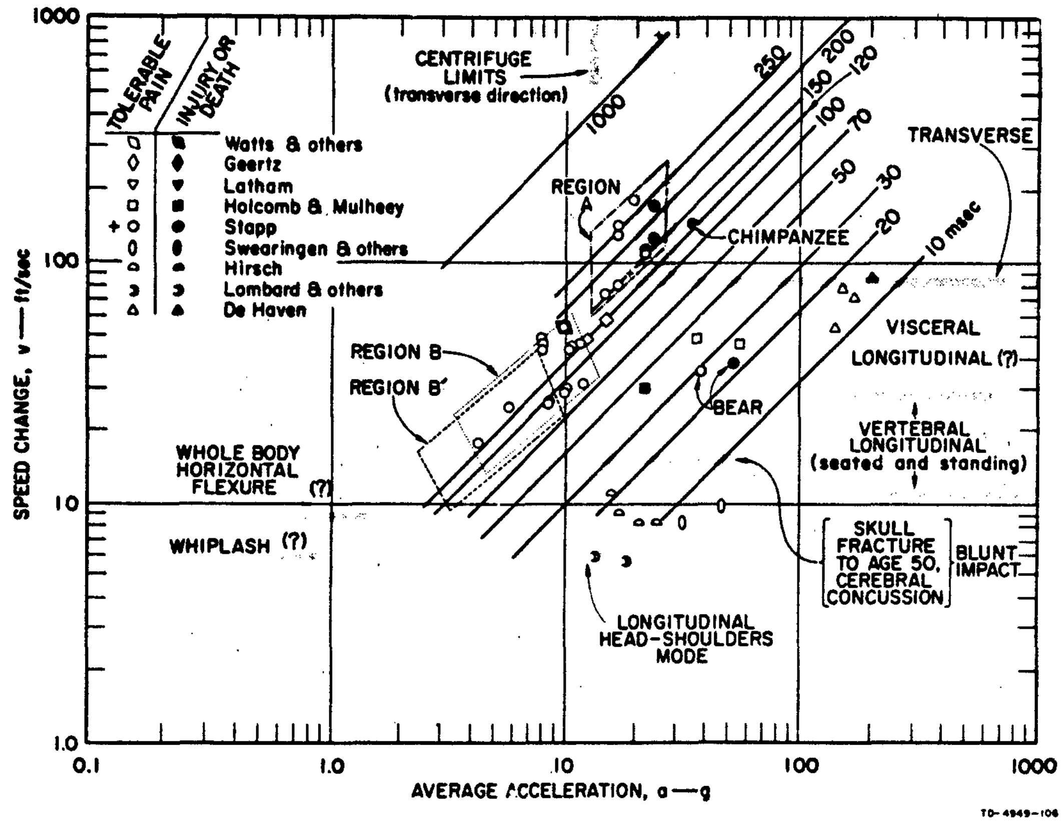

FIG. S-1 HUMAN TOLERANCE LIMITS TO IMPACT

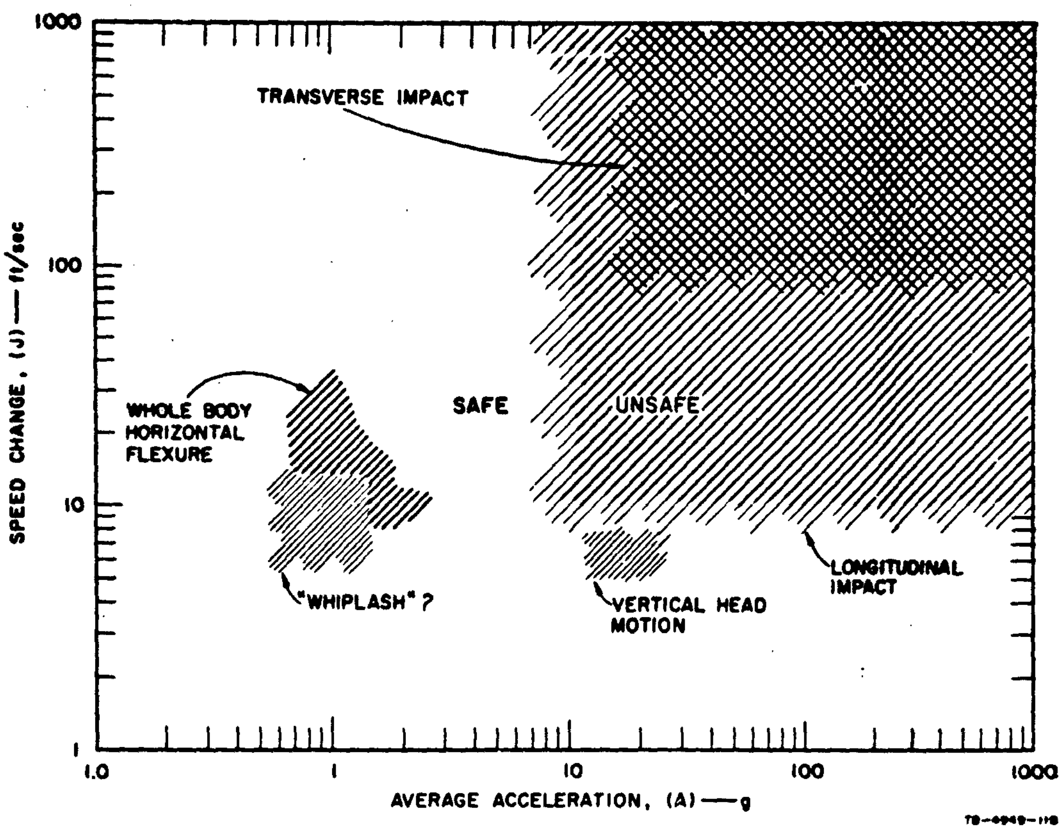

FIG. S-2 HUMAN TOLERANCE TO IMPACT

"Vertical head motion" would be aroused by either a blow to the top of the head or an upward or downward thrust of the shoulders.

Major, whole body limits are: in the longitudinal mode, speed change (V_o) 8 to 10 ft/sec and average acceleration (A_o) 7 to 10 g, and in the transverse mode, $V_o \approx 80$ ft/sec and A_o between 15 and 20 g. All important bodily frequencies appear to be in the range from 1 to 30 Hz.*

Data gathered in Fig. S-1 are outlined in the following paragraphs.

1. <u>Centrifuge</u>

Man's tolerance to centrifuge acceleration in the transverse direction is limited at an average acceleration between 10 and 15 g by a

* Hz = Hertz = unit now used for cycles per second.

subsystem which is not important in his reaction to ground motion (Ref. 11) but it does establish a lower bound pertinent to other subsystems on the sensitivity curve.

2. Sled Impacts

More closely pertinent to a study of the effect of ground motion are the precisely monitored impacts of seated men on sleds carried out by Stapp and his co-workers (Refs. 12-14). In two series of experiments they explored the effects of accelerations lasting between 70 and 260 msec as well as the effects of body orientation with respect to the acceleration vector. For a chest-forward or back-forward impact their limits appear to lie in the range of average accelerations between 20 and 25 g (Region A in Fig. S-1). Because of the existence of a strong component of acceleration along the body axis, reclining in their seats aboard the sled were much less tolerant and their limit appears to have been in the neighborhood of 7 to 10 g average acceleration. (In horizontal coordinates data from this series appear in Region B of Fig. S-1. When corrected for the inclination of the subject, data fall in Region B'.) No fully side-on impacts were performed but rotations up to 45^0 of impact direction seemed not to change tolerance limits. Studies of upward motion from an ejection seat by Latham (Ref. 15), Watts and others (Ref. 16), and Geertz (Ref. 17) demonstrate tolerance levels in the same area of the duration axis as explored by sled impacts. Dropping a volunteer strapped inside a capsule, Holcomb and Mulheey (Ref. 18) produced spinal injury with longitudinal an average acceleration of 22 g, a speed change of 30 ft/sec and a duration of 42 msec.

3. Impact of Feet and Buttocks

Higher average accelerations along the longitudinal body axis can be tolerated if pulse duration is reduced, as has been demonstrated in upward impacts by Hirsch with his ship-shock simulator (Ref. 19) and Swearingen, McFadden, Garner, and Blethrow with free falling, standing, and sitting subjects (Ref. 20). In these experiments durations of the main impact pulses varied from 6 to 21 msec. Standing men holding their knees locked and upright men seated in a hard chair will accept speed changes (impact speeds) between 8 and 10 ft/sec at average accelerations

from 15 to 47 g. Bending the knees or using chair padding increase the tolerable acceleration by factors of approximately 2 and 4, respectively. Whether the acceleration vector is up or down does not make much difference to the observed tolerances.

4. Transverse Impact in Free Fall

Accidental or suicidal broadside body impacts of estimated 12 to 18 msec duration have been quantitatively evaluated by DeHaven (Ref. 21). Acceleration vectors were transverse to the spinal axis directed mainly fore-and-aft or back-to-front. When the struck surface has been relatively uniform and yielding, impact speed has been as high as 80 ft/sec without producing injury. According to DeHaven's observations speeds above 80 ft/sec under similar impact conditions have led to death. Holcomb and Mulheey did not injure their encapsulated volunteers with speed changes of 50 ft/sec transverse to the body axis taking place in 25 to 35 msec (Ref. 18). One of their subjects landed on his side and two of those studied by DeHaven suffered sidewise components of motion.

5. Harmonic Model of Whole Body

These data are consistent with the action of two independent failure modes in the human body as a whole, one effective in longitudinal motion, the other in transverse. The speed change asymptote V_o making up part of the sensitivity curve corresponding to the transverse failure mode lies near 80 ft/sec. The acceleration asymptote A_o must be established by the occurrence of a much lower degree of injury at about 17 g. The "natural period" T_n of such a mode based on Kornhauser's analysis (Ref. 35) probably lies between

$$T_n \leq 4 \frac{80}{17 \times 32} = 580 \text{ msec}$$

and

$$T_n \geq 17 \frac{\pi}{2} \frac{80}{17 \times 32} = 230 \text{ msec}$$

the range of values reflecting the possible influence of acceleration pulse shape.

There could in principle be two separate longitudinal failure modes, depending on whether the acceleration were transmitted principally through locked knees or through the buttocks. If so, the two V_o asymptotes seem to lie close to each other in the range 8 to 10 ft/sec, determined by voluntary tolerance. There are no data establishing an A_o asymptote for the standing posture; for seated men the acceleration limit seems to lie in the range from 7 to almost 15 g. Since Stapp's work is the best documented in the open literature, his result of approximately 7 g average acceleration will be used to calculate a natural frequency f_n for the seated longitudinal failure mode:

$$14 \text{ Hz} \leq f_n \leq 7.1 \text{ Hz} \quad .$$

Dieckmann (Refs. 10 and 22) concludes from his measurements of the dependence of impedance and its phase angle upon frequency that both the seated and standing man in longitudinal vibration can be represented by two coupled simple harmonic oscillators, which in the seated posture have frequencies near 4 and 30 Hz. Coermann, Ziegenrucker, Wittwer, and von Gierke (Ref. 23) find a purely visceral mode of longitudinal oscillation at 3 Hz. All observations agree in pointing to the region 1 to 30 Hz as particularly important for people in motion parallel to the spine. The accuracy of the impact data plotted in Fig. S-1 probably does not allow a fine resolution of body frequencies.

Dieckmann also provides standards for tolerance to steady, sinusoidal vibration and these are consistent with those of Fig. S-1 interpreted as the sensitivity diagram of a two-(coupled)-degree-of-freedom oscillator except at frequencies in the range from 20 to 30 Hz (corresponding to durations equal to roughly 16-9.5 msec, in Fig. S-1.

6. Vertical Motion of Head

Dieckmann's data suggest there may be a controlling failure mode in the head-neck subsystem which conceivably could reduce the tolerance asymptote V_o in Fig. S-1 for pulses of duration less than 10 msec. His tolerance limits in this frequency regime can be reconciled with those set by Lombard, Ames, Roth and Rosenfeld (Ref. 24) for downward blows to the top of the padded heads of volunteers.

7. Whiplash Motion of the Head and Neck

Relative transverse fore-and-aft displacement of the head with respect to the body has been studied with steady sinusoidal oscillation by Dieckmann (Ref. 25) and under impact of the head by Lombard, Ames, Roth, and Rosenfeld (Ref. 24). Results are compatible with a simple harmonic mode of frequency between 1 and 2 Hz, maximum tolerable spring excursion of about 6 inches, V_o = 6 ft/sec and A_o = 0.6 g (Fig. S-1). Since the location of the site of the limiting pain was not explicitly mentioned by Dieckmann and the importance as a limiting factor of head pain as well as neck pain in the Lombard experiments is clear, the "whiplash" thresholds cannot be considered well established.

8. Whole Body Bending

Observations by Dieckmann (Ref. 25) in steady sinusoidal horizontal oscillation of standing subjects suggest a fundamental flexural mode between 1 and 2 Hz; a body bending as a half wave length then has a frequency one half this, which for theoretical reasons Dieckmann chooses as 1.4 Hz. Maximum tolerable relative displacement between head and feet equal to 1 ft implies V_o = 8.8 ft/sec, $A_o \cong 0.6 - 1.2$ g. There are no good tolerance observations to confirm this estimate.

9. Tabulation of Bodily Harmonic Behavior

Response modes outlined above are collected in Table S-1. Frequencies shown are consistent with the bounds

$$\frac{1}{4}\frac{A_o}{V_o} \leq f_n \leq \frac{2}{\pi}\frac{A_o}{V_o} \qquad ,$$

but whenever possible stem from impedance measurements. As will be seen later, such values do not necessarily correspond exactly to frequencies of component subsystems but are the modal frequencies of the complex (linear) system. There is one such frequency for each subsystem.

Table S-1

HARMONIC BEHAVIOR OF THE HUMAN BODY

Failure Mode	Speed Change V_o (ft/sec)	Average Acceleration (g)	Modal Frequency (Hz)
Longitudinal, seated	11	7	5
Longitudinal, standing with knees locked	10	--	--
Longitudinal, lying with skeleton checked	45 (?)	> 7 (?)	3.5
Longitudinal, acceleration of head	6	15	25
Transverse, whole body	80	17	1.7 - 2.3
Transverse, whiplash of head	6 (?)	0.6 (?)	1 - 2
Transverse, flexure of whole body (sagittal and frontal planes)	8.8	0.6 - 1.2	1.4

B. Human Resistance to Shock Waves

Breakage of foot bones when standing men are impacted from below can be understood reasonably well as a shock wave phenomenon using values of bone tensile strength and bone shock impedance reported by Goldman and von Gierke (Ref. 26). Black, Christopherson, and Zuckerman (Ref. 27) as well as Durcovic and Hirsch (Ref. 28) have established observationally a threshold for heel fracture at 10 ft/sec. This value very likely decreases markedly with increasing age above 50 years (Ref. 29). Skull fractures appear to be more complicated, involving flexure of a skull region; empirically the fracture threshold has been placed at 17 ft/sec by some investigators (Ref. 4) but the possibility of departure from this level when impact conditions are varied does not seem to have been clearly ruled out.

When man is in the sitting posture, upward impact threatens not the pelvic bones but the vertebrae. Fracture of the backbone has been analyzed as resulting from pinching in flexure and is thus not treated as a shock wave phenomenon.

The mechanism of an incapacitating brain injury is at present still conjectural. By analyzing a small sample of automobile accident injuries, Swearingen (Ref. 30) puts the threshold below 20 ft/sec; and Lissner and Gurdjian infer a threshold near 15 ft/sec from a study of minimal skull fractures in cadavers (Ref. 31). There are no data establishing the presence or absence of age dependence. The impact conditions to which this threshold, viz., 15 ft/sec, applies are similar to those likely to be suffered by people falling against floors or walls in underground shelters.

C. Description of the Motion Environment

Knowledge of the free field motion environment to which underground shelters may be exposed comes almost wholly from observations during surface explosions of nuclear weapons at the Eniwetok Proving Ground and during above-surface nuclear detonations at the Nevada Test Site. Because these two locations are quite different geologically, similarities in ground motions at the two sites are looked upon as very likely general features of nuclear-induced soil response everywhere.

1. Ground-Transmitted Motion

One such feature has been described by Sauer (Ref. 1) as a strong rolling motion with period between roughly 0.1 and 10 sec which may appear nearly coincident with or ahead of the airblast and which is probably the result of energy transmission through an underground layer of high seismic velocity. In Nevada because of the small yields used it is not detected or is not significant at ranges where peak airblast overpressure is greater than about 15 psi; at Eniwetok it dominates the motion at all ranges so far investigated. In addition to its period, which increases with range, this component can be characterized by the peak speed achieved in the ground moving under its influence. Both in Nevada and at Eniwetok, peak speed falls inversely with the square of radius from ground zero but the peak speed in Nevada is approximately four times larger than that at Eniwetok at the same range scaled to 1 kt. Strengths of horizontal (radial) and vertical components of this motion are nearly equal and do not diminish appreciably between the

surface and depths of 30 ft or more. Quantitative features of the wave are not understood theoretically.

2. Airslap Motion

Much better understood is soil behavior immediately under the influence of the airblast front. Near the surface, elementary theory of one-dimensional shock waves produces values of peak downward ground speed and peak elastic displacement in good agreement with observations (Ref. 1). As expected, the downward ground speed under airslap follows generally the intensity of the overpressure on the surface. (Absence of a clearly distinguishable airslap pulse at Eniwetok has been attributed to the high water table which immediately returns a cancelling reflection to the surface.) It is reasonable to assume that ground speed under airslap varies from site to site inversely as the shock impedance of the soil.

Theoretical understanding of the decline of airslap induced speed and elastic displacement with depth and of the relation among air pressure, and soil and ground acceleration is not well developed. Energy loss to the soil plays a role in determining attenuation of both speed and acceleration. Presumably, relatively incompetent Nevada Test Site playa has extremely strong loss mechanisms, but even so, under megaton explosions downward speed loses less than half its surface value by 50 ft. Fragmentary observations under airslap in limestone point to peak accelerations approximately twice the highest seen in Nevada under the same peak overpressure. Horizontal speeds due to airslap in Nevada can be as great as one-half the vertical.

3. Conservative Response Spectra

Clearly, the unknowns appear formidable in a description of nuclear motion environment. However to the questions suggested above there are generally physically reasonable, conservative answers:

- Peak acceleration in hard rock may be assumed to be five times the average in Nevada playa
- Horizontal airslap motion is at most equal to the vertical
- Attenuation of all ground motion with depth can be ignored.

Unfortunately there are further questions which cannot be handled so easily at this time:

- If airblast or crater energy can be collected by a fast layer for remote delivery to the surface, can this energy ever be concentrated or focused? (Sauer, Ref. 1)
- What role might the violent Rayleigh wave play in high speed materials? (Merritt and Newmark, Ref. 7).

For now, these two questions must be set aside. Effects of strong ground motion on people and equipment will be derived from response spectra computed from observed waves and from reasonable extrapolations thereof. Some account will be taken of the possibility of modifications in these spectra arising from behavior of the shelter structure although this complex subject needs further study before any modifications can be definitely described.

The purely airslap waveform of vertical motion leads to a very simply described undamped response spectrum of the kind which shows the product of circular frequency (ω) and maximum spring distortion (x_M) on the ordinate against circular frequency on the abscissa. Both coordinates are in logarithmic scales. Spectra of airslap motion are then very closely enveloped or fitted by trapezoids defined by peak ground displacement, 1.5 times peak ground speed and twice peak acceleration in the wave, in the way set forth by Sauer (Ref. 1) and by Merritt and Newmark (Ref. 7). Figure S-3 shows two such airslap spectra intended as opposite extremes: one is computed from motion near the surface of Nevada playa and the other is derived from estimates of the response of limestone. Origin of motion in both cases is supposed to be an airblast wave of 50 psi peak pressure. The figure suggests that as far as airslap is concerned, exposure in Nevada Test Site soil provokes almost the highest response possible in systems with frequencies in the range 1 to 10 Hz. Extremely hard soils, on the other hand, do not become more hazardous than NTS soil under airslap until system frequency exceeds 500 Hz. Conservative practice might then extend the Nevada Test Site envelope as indicated by the dotted lines in the figure; all airslap spectra at 50 psi peak airblast overpressure will be found within these bounds.

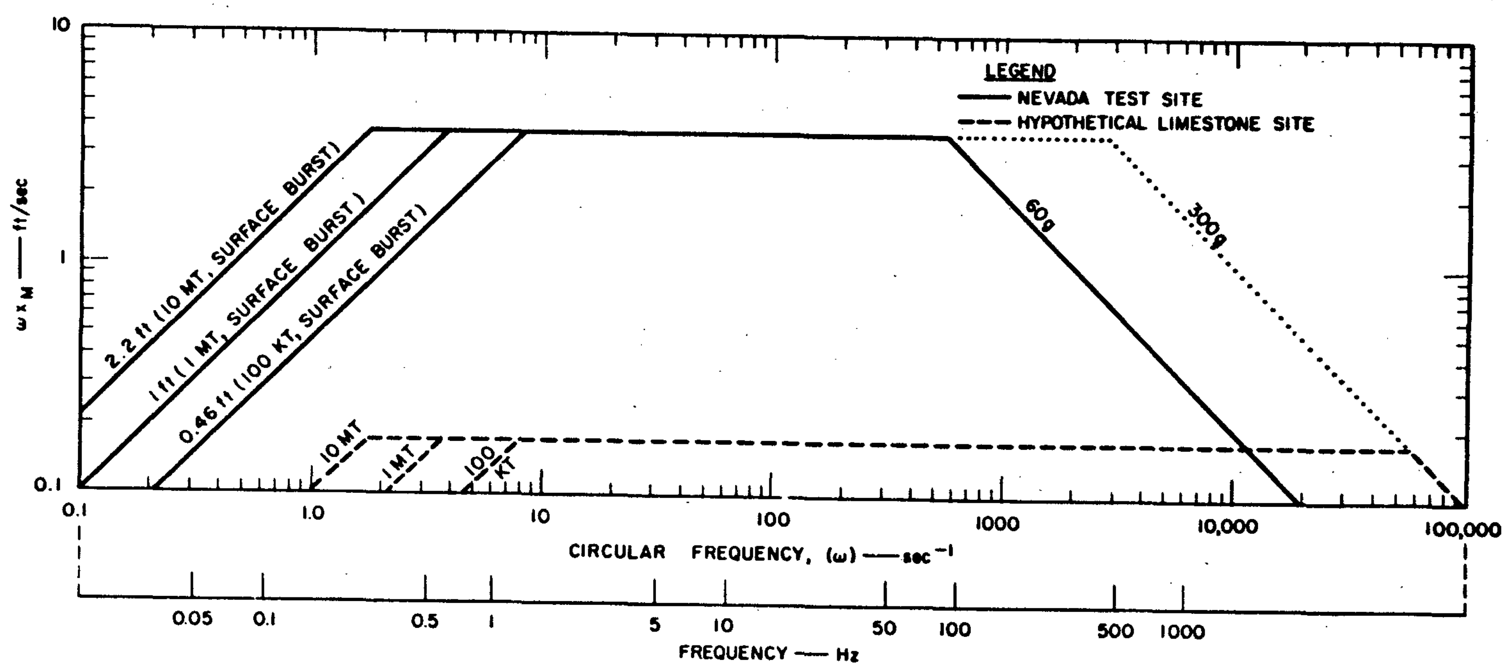

FIG. S-3 VERTICAL RESPONSE SPECTRAL ENVELOPES FROM A 50-psi AIRSLAP GROUND MOTION

However, in soils with very hard layers near the surface, airslap may not be the dominant cause of ground motion at the 50 psi range; airslap will be superimposed on the rolling ground-transmitted wave. At present, which of the two motions will be dominant cannot be determined surely—even with rather detailed geologic information. As pointed out earlier, however, the shape of the ground-transmitted is fairly certainly known empirically. An idealization of this shape taken from Ref. 1 is pictured in Fig. S-4 in coordinates of speed against time. The characteristic time T_2 depends in a complicated way on yield and geology (Ref. 1). For a given environment at a given overpressure, T_2 increases with yield. Duration of the whole wave is apparently 2.5 T_2. Without a priori knowledge of the geologic environment, the only limit on T_2 available from data so far collected is an upper bound, viz.,

$$T_2 \text{ (msec)} \le \frac{R}{4} + 100$$

where R is the distance in feet from ground zero.

As might be expected, the response to the rolling motion is strong over a narrow frequency range. Figure S-5, a recalculated version of a figure in Ref. 32, is the nondimensional spectrum in linear coordinates of the idealized wave shown in Fig. S-4. Units of ordinates are peak speed V_M in the wave; units of abscissas are total durations, i.e., 2.5 T_2. Peak value of ωx_M is $\sim 5V_M$ which occurs only near a frequency $f = 1.6/T_2$. For a surface burst, peak airblast overpressure equals 50 psi at approximately 500 ft/kt$^{1/3}$ so that the sensitive frequency f can be bounded:

$$f \ge \frac{1.6}{\frac{0.500}{4} W^{1/3} + 0.1} \text{ Hz}$$

where W is yield in kilotons. Sauer's formula suggests that, wherever ground-transmitted motion is detectable, $T_2 > 100$ msec. It is not clear that waves of this characteristic time have actually been seen but the formal lower bound puts an upper bound on f

$$f < 16 \text{ Hz} \quad .$$

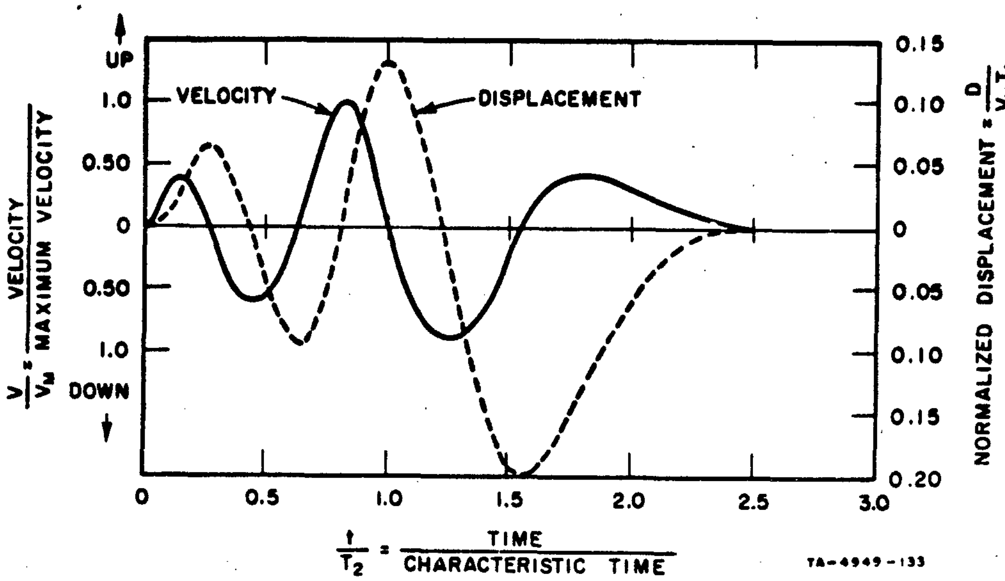

FIG. S-4 IDEALIZED GROUND-TRANSMITTED WAVE (After Sauer, Ref. 1)

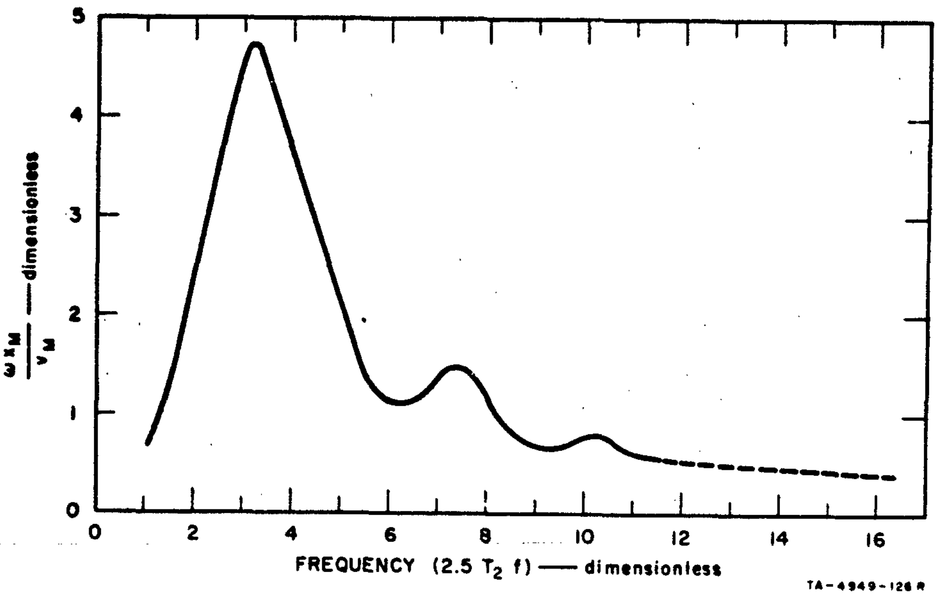

FIG. S-5 RESPONSE SPECTRUM FROM GROUND-TRANSMITTED MOTION

For various yields then:

	100 kt	1 mt	10 mt
f >	2.3 Hz	1.2 Hz	0.57 Hz
f <	16	16	16

Since the frequency range defined above coincides with most of the region of high sensitivity in the human body, ground-transmitted motion may be

an unusual threat to the body. Because maximum values of acceleration in this wave are 5 g or less, the spectrum falls off very fast above 20 Hz in all cases.

Horizontal and vertical components of the ground-transmitted wave are essentially alike.

Several examples illustrating the possible reduction of peak spectral ordinate by as much as a factor of 2 -- presumably due to destructive interference -- when the two basic idealized waveforms for vertical motion are algebraically added have been derived as a part of this project and appear in Chapter V-E. The waveforms analyzed were synthesized in an attempt to portray realistic attack conditions and the results no doubt correctly indicate the most probable direction of change due to superposition. Theoretical justification for simple addition of motions, however, is not clear.

A parameter study reported by the Ralph M. Parsons Company (Ref. 32) in which spectra for various arbitrary combinations of idealized airslap and rolling components were computed suggests peak response to simultaneous airslap and roll may be as high as $4V_M$.

To complete this description of ground motion environment at the 50 psi range, only a value for V_M in the ground-transmitted wave is lacking. An incontestable value of 2 ft/sec can be drawn from Eniwetok experience. However, it hardly seems conservative to overlook the motion beyond 15 psi at the Nevada Test Site, which is stronger than motion at comparable ranges at Eniwetok and which often shows the intensity of rolling motion amounting to more than half the airslap intensity at the same range. Heuristic arguments can be made to justify extrapolation of the law relating peak speed to range in Nevada back toward ground zero from the 15 psi radius where the ground-transmitted motion first becomes important. The extrapolated value of V_M is 8 ft/sec, i.e., four times the "proven" intensity. It is a reasonable speculation that the maximum ordinate ωx_M of the response spectrum due to ground-transmitted motion may reach as high as 32 ft/sec somewhere in the frequency region from 0.5 to 16 Hz in some geologic environments.

D. Evaluation of the Motion Threat to People

Analysis of the hazard to people in underground shelters will depend on whether the individuals are free or restrained and whether the motion is pure airslap or not.

There is no significant hazard due to a 50 psi shock wave transmitted from the ground through the shelter structure to an individual inside.

1. Restrained Shelter Occupants

"Restrained" means that the torso of each individual moves with the part of the shelter structure to which he is attached. The response spectrum associated with the motion of this part can furnish a measure of the degree to which the motion disturbs a simple harmonic system so attached; but the human body has at least two degrees-of-freedom coupled to one another. Since Dieckmann's data (Ref. 10) suggest that the two major bodily subsystems may have masses and frequencies in the ratio of 2 or 3 to 1, coupling cannot be ignored in analyzing the reaction of either one to ground motion. It is shown elsewhere in this report that while coupling of such subsystems tends to reduce the area of tolerance in the sensitivity diagrams of both subsystems below that of the uncoupled system, the shape of each diagram is only slightly distorted around the juncture of the two asymptotes. (This is demonstrated for rectangular acceleration pulses and assumed for other shapes.) Thus the overall sensitivity diagram for a composite system of this kind will be a superposition of the curves of the usual shapes, as illustrated schematically in Fig. S-6. Furthermore, although no one of the component curves will occupy exactly the same location it would were the system alone (and in fact the apparent "natural frequencies" determined by the meeting points of the asymptotic pairs may be shifted), it is part of the basic assumption under which response spectra are used to compare the effects of various complex motions upon complex systems that the response spectrum ordinate can be computed from the speed asymptote V_o by the formula

$$\omega x_M = V_o \quad .$$

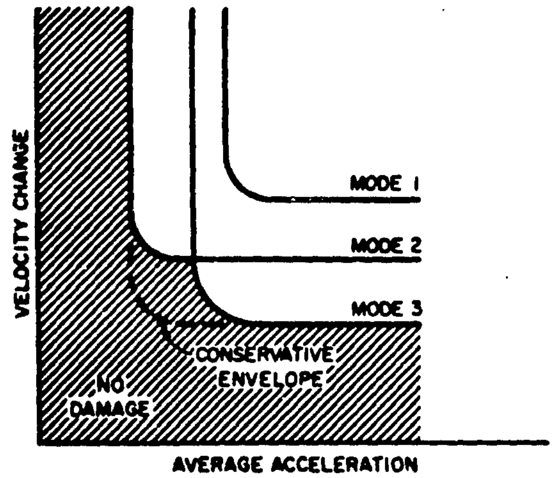

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FIG. S-6 SCHEMATIC DIAGRAM OF SENSITIVITY CURVES FOR SYSTEM WITH SEVERAL MODES OF FAILURE (After Kornhauser, Ref. 9)

The frequencies ω in the formula will be the "apparent" values from the observed sensitivity curves. Tolerance or acceptance of a motion represented by a response spectrum means the entries for V_o in Table S-1 are greater than the ordinates of the spectrum over the frequency range stated in the last column of Table S-1. (Were good quantitative mechanical analogs of the human body available which showed all the important features of the body's reaction to motion, a better procedure would be to calculate the response of the analogs to typical waveforms.)

Figure S-3 makes it clear there is little threat to restrained men in the airslap motion. Highest ordinates (which occur in the frequency range approximately from 1 to 50 Hz) are below 4 ft/sec; for the two major longitudinal modes (rows 1 and 2, Table S-1) V_o is 10 or 11 ft/sec. In the highly unlikely event the horizontal airslap spectrum equals the vertical, there is a weak threat to the head and neck in whiplash since V_o in that mode is tentatively set at 6 ft/sec and subsystem frequency is between 1 and 2 Hz. If the soil is Nevada-like and the peak ground acceleration is above 10 g, there may be a similar weak threat to torso-restrained men due to stretching of the neck (row 4, Table S-1).

Horizontal and vertical spectra of the ground-transmitted motion are nearly equal and the proven spectral maximum can be as high as 3 × 2 = 6 ft/sec. The parameter study (Ref. 33) suggests a maximum at the 50-psi range equal to 4 × 2 = 8 ft/sec. At the 50-psi range in Eniwetok, the frequency corresponding to this peak is 4 Hz but only a slight deviation of characteristic time T_2 is needed to make the whiplash mode dangerous. Even the seated, longitudinal mode (V_o = 11 ft/sec) is close to hazard. (Dieckmann [Ref. 10] reports a maximum in impedance for seated vertically oscillated men at 4 Hz.) However, if the speculative level of ground-transmitted motion is used, i.e., $\omega x_M \cong 4 \times 8 = 32$ ft/sec, strong threats to restrained men in both the seated and standing postures arise especially when T_2 is near 0.4 sec at the 50-psi range.

The frequency of the vertical oscillation of the head with respect to the shoulders is probably too high for involvement in ground-transmitted motion (Dieckmann finds a resonance between 20 and 30 Hz).

With one exception structural resonances calculated by Agbabian-Jacobsen (Ref. 33) for underground shelters fall in the frequency range above 20 Hz and pose no additional threat to the inhabitants. It is not clear that this conclusion is generally true of all possible structures, however.

2. Unrestrained Shelter Occupants

The difference between the hazards to unrestrained and to restrained people arises from the likelihood that an unrestrained body will lose contact with the floor only to have that contact later violently re-established. Two possibilities are taken up separately: normal posture is kept or toppling occurs.

Without toppling airslap is harmless. The body will overtake the falling floor usually before the floor finds its lowest level and the relative speed of collision can easily be shown to be less than 6 ft/sec, which is below the tolerable level for both seated and standing men. Horizontal impact speed between a wall and a man pressing against it is at most 3 ft/sec.

Under ground-transmitted motion of "proven" intensity, i.e., a peak vertical speed of 2 ft/sec, a person's free flight without toppling will cover at most a few inches. At the speculative intensity level, i.e., peak speed equal to 8 ft/sec, to avoid danger, people would often have to unlock their knees if standing or seek support from arm rests if seated. Recollision speeds between the floor and the body are very likely to be 8 ft/sec and could be as great as 15 ft/sec for a certain narrow range of unfavorable values of characteristic time T_2. For most adults in good health a threat of this kind would be of only marginal seriousness, but these impact speeds may be hazardous to the very old or very young or the sick.

3. Toppling

Toppling is a body's loss of its normal orientation with respect to its surroundings resulting in a violent fall or collision of a sensitive part of the body with a rigid structure. Any standing man who is toppled onto a hard surface runs the risk of broken bones, dislocated joints, or brain injury. For example, a free fall through 5 ft results in an impact speed of 18 ft/sec which is above the threshold for head injury. A man six feet tall standing stiffly as he topples without slipping may strike his head on the floor at a speed of 24 ft/sec.

The likelihood of toppling from the standing position is very great. The floor will be moving both vertically and horizontally at once. According to Crede (Ref. 32) the frictionally maintained connection between floor and feet will be essentially lost during any downward motion with acceleration greater than 1/2 g and even the rolling ground-transmitted motion usually exceed this level at the 50-psi range.

There are no data from which to draw inferences as to the likelihood of successful defensive efforts by toppled people to prevent injury. There is time enough during a free fall of 5 ft to rearrange the body for a favorable impact but whether all people would take proper advantage of the time under conditions of a nuclear attack is not clear. It would seem wise to seat as many people as possible in chairs tied to the floor and to provide these chairs with arm and head rests. Since all people

cannot be seated all the time, the further precaution of covering floors and walls with a fairly thick layer of low shock impedance material should reduce the hazard from toppling to an insignificant level. Some kind of open netting might be hung from the ceiling to provide standing people a means of breaking their fall.

E. Evaluation of the Motion Threat to Equipment

Both because of the great range occupied by important equipment and because of the relative lack of interest, equipment response to motion and shock has not been reported so fully nor in such detail as the information outlined above for people. Testing consists of impacts of whole pieces of equipment and the only procedure available for analysis is a comparison of response spectra of laboratory impacts with likely environmental spectra, as far as the latter can be forecast.

Although close envelopment of almost any desired spectrum is readily possible with combinations of drop and shake tests, the only tests used extensively have been hammer blows or, to a less extent, underwater shock waves. These are embodied in the three famous U.S. Navy shock tests, delineated in MIL-S-901C. For smaller equipment a U.S. Air Force drop test generally more suitable in duplicating ground motion spectra has sometimes been used, MIL-S-4456; in this and related tests the arresting mechanism can be tailored to produce spectra of a wide variety.

Generally hammer blows create overly strong high frequency components (above 100 Hz) and insufficiently strong low frequency components (below 20 Hz). Most equipment has such little responsiveness to low frequencies that their absence in test motion is not important but there are notable exceptions, such as cabinets carrying relatively heavy components, certain large mounts for heavy machinery, and large wet cell batteries. Also, thought is often not given to the possibility of the existence of resonances in the shelter structure itself which might amplify ground motion dangerously in the low frequency regime below 50 Hz where many large pieces of equipment have major resonances. Specifications for shock testing often ignore the rolling component of motion as contrasted

to the more familiar airslap. For much equipment the most taxing motion in the field will be an upward going tensile wave which is in all tests simulated by a downward or sometimes an upward moving compression. Simultaneous excitation of longitudinal, transverse, and rotary modes is rarely achieved and the effects of coupling between them remains largely unexplored. Thus, even a "passing" mark on a piece of equipment cannot always be accepted as valid for all shelter sites without doubt or qualification.

Considerable testing has been inspired by shock resistance requirements laid down in structural specifications for the large number of underground Civil Defense command and control centers which have been designed and built in the past decade. Other data collections--not quite as pertinent as the testing of Civil Defense shelter equipment--have come out of the experience of the U.S. military forces. (Some of these two kinds of data are described in more detail in the body of this report.) In general, shelter equipment can be put into two roughly equally populated categories: that which has undergone some sort of shock test and that which has been assigned by its manufacturer a fragility level below that needed for the nuclear ground motion environment at 50 psi peak airblast overpressure. The basis for the manufacturer's assignment is seldom absolutely clear but it appears to be generally accepted as evidence of the need for shock isolation of the items concerned. The first category can also be broken down into two very roughly equal sized categories: that equipment which passes the tests without modification and that which must be modified. Whether the failures can be attributed to the excessive severity of the test or whether the equipment which passes the testing would pass an adequate test is never clear. The only conclusion that can be drawn from the large number of test results so far available is that continued isolation of many items is necessary until proof to the contrary is developed.

It seems reasonable to expect that equipment resistant to ground motion without isolation could be designed. Since World War II the U.S. Navy subjected all new equipment destined for shipboard mounting to shock testing in order to make it resistant to nearby underwater

explosions. Stock items very often fail and must be modified--sometimes more than once--until the test is satisfied.* Generally these tests are more severe in the frequencies dangerous to equipment than is nuclear ground motion.

Test results so far are generally encouraging to the view that with slight modification much equipment could be hard-mounted. Small units of electronic equipment are regularly made shock resistant; lighting fixtures have been found resistant; but testing has shown rotating machinery also can withstand the airslap environment: several 50-100 HP electric motor-pump sets, a 580 BHP diesel generator combination as well as smaller motors, pumps, and transformers have all withstood shock testing of a magnitude approaching that needed for the 50-psi airslap environment. Unlike people, equipment can be modified to meet changing requirements. But it is certainly too early to dispense with continued and improved testing.

* K. W. Rosenthal, West Coast Shock Test Facility, San Francisco Naval Shipyard, private communication.

IV OUTLINE OF ANALYTIC METHOD

A. Nature of Injury

A mechanism is injured when a part or parts are displaced with respect to one another so that the whole can no longer function together. The physical scale of the injury can be microscopic or macroscopic. In addition, human beings subjected to impact in underground blast and radiation shelters must not only be able to function but should be kept free of all but transitory pain.

The distinction between the microscopic and macroscopic scales is somewhat arbitrary. Injuries which can be understood better as the result of the transmission of shock waves in the body are treated separately from those which can be thought of as stemming from dislocations of major body parts. Thus, skull fracture is "microscopic" and the neck sprain known as whiplash is "macroscopic." Spall of metal from vital equipment parts is "microscopic"; bending an axle is "macroscopic." The former involves relatively short durations and high frequency components of the motion; the second is associated with long-lasting, low frequency motions.

B. A Simple Model

If dislocated parts return very quickly to their proper configuration, there is no injury. Since an elastic system is one which also recovers its shape in this way, a very useful and convenient model to keep in mind during a discussion of impact damage is the mass and spring on a base. There must of course be a dissipative mechanism or dashpot to make the scheme realistic, but generally the exact actual degree of dissipation will not be known. The analog of injury is over-compression or over-stretching of the spring, i.e., distortion beyond a point where recovery is possible. Since most of the objects the shelter contains will be likely to impact are very massive, viz., walls, floors, or equipment rigidly attached to walls and floors, the compression or stretching of the spring in the model

should be thought of as the result of a specified displacement of the platform or base. Generally, the visco-elastic analog of an object or person must contain a number of coupled subsystems.

C. Sensitivity Diagram

As long as the forcing function applied to the base of a single harmonic oscillator is a simple one-sided* pulse of acceleration, maximum spring distortion can be defined by a curve of simple shape in certain special logarithmic coordinates. An example adapted from Kornhauser (Ref. 8) is shown in Fig. 1, where abscissas are average acceleration and ordinates are speed change of the base resulting from the pulse. The defining curve consists of two parts, one of which is a horizontal straight line paralleling abscissas greater than a certain least value and the second is a more complicated curve whose shape depends on pulse shape but whose location is for a wide array of pulses always found within a vertical band such as that indicated by crosshatching in Fig. 1. The ordinate V_o at the horizontal line is, if the oscillator is undamped, simply

$$V_o = \frac{2\pi x_M}{T_n}$$

where x_M is the maximum spring distortion and T_n is the natural period of the oscillator. As pulse duration increases, the second part of the defining curve becomes a vertical straight line at abscissa A_o which is bounded as follows:

$$\frac{\pi}{2}\frac{V_o}{T_n} \leq A_o \leq 4\frac{V_o}{T_n} \quad .$$

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* Pulses without a change in the direction of acceleration. Strictly such pulses seldom exist in practice but the influence of a slow return to original speed is slight.

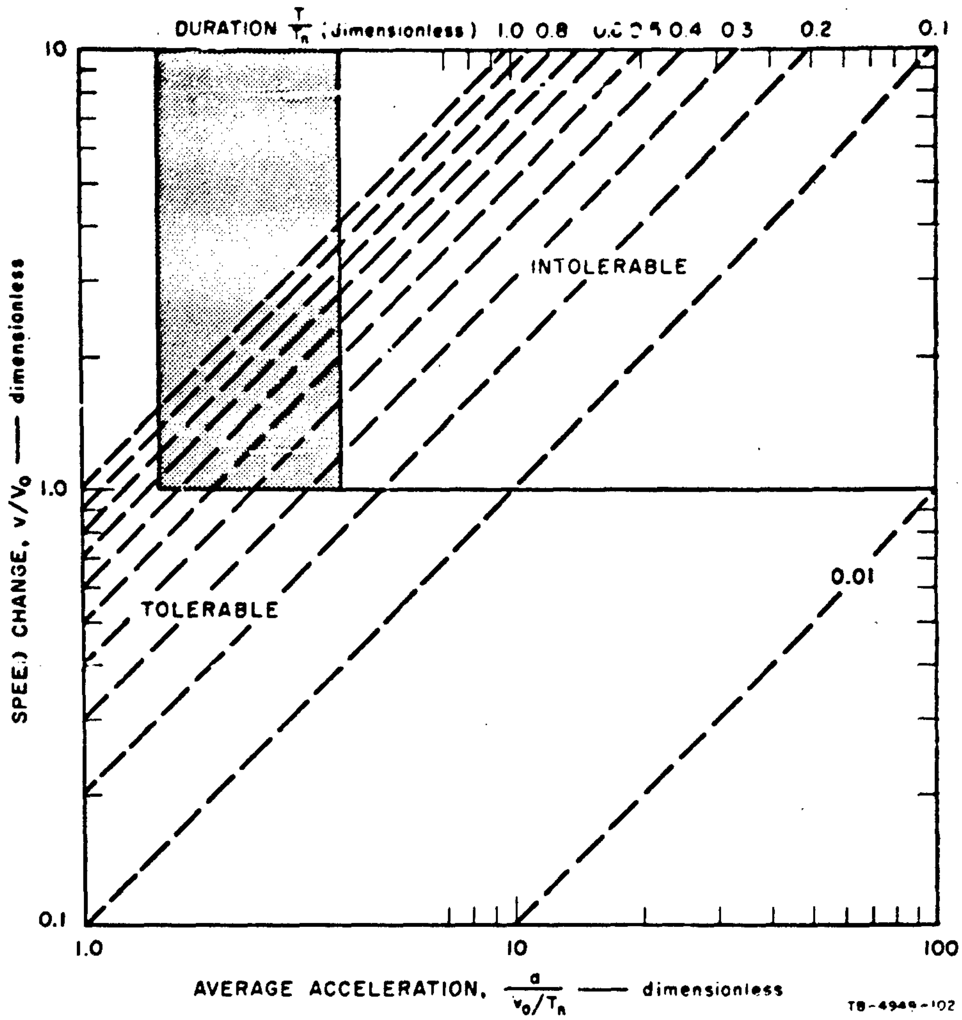

FIG. 1 REGION OF INTOLERABLE STRAIN OF SIMPLE OSCILLATOR DUE TO ONE-SIDED ACCELERATION PULSE

Pulses with coordinates lying both above and to the right of the defining curves produce spring distortion greater than that defined and points below and to the left are associated with lesser distortion. Thus when Fig. 1 is used to define sensitivity, the upper area is labeled "intolerable," the lower, "tolerable."

In the coordinates of Fig. 1, loci of constant pulse duration are lines of slope +1. The juncture of the two parts of the defining curve occurs when pulse duration is between 0.3 T_n and 0.7 T_n.

Further discussion of the sensitivity diagram including certain mathematical derivations can be found in Appendix A and in Ref. 8.

The diagram is useful because many of the observational data on humans stem from impacts which are essentially one-sided acceleration pulses of varying lengths. As apparently Kornhauser (Refs. 8, 9 and 34) was the first to report, observed animal injury thresholds do trace out a defining curve in the coordinates of Fig. 1 like that described above; that is, approximate values of V_o, A_o, and T_n for an equivalent harmonic oscillator can often be read directly from data obtained during test impacts. Sensitivity of the living system to untested motion whose response spectrum is known can then be inferred from V_o and T_n.

D. Effect of Damping on the Sensitivity Diagram

The presence of damping influences the values of V_o and T_n but for amounts of damping likely to be found in the human body the effect is not large, as demonstrated in Table I:

Table I

REDUCTION OF CRITICAL SPEED (V_o) DUE TO DAMPING

Percent Critical Damping	Percent Change in T_n	Maximum Percent Change in V_o
5	1	7.6
10	2	13
20	5	25
30	5	33

Damping alters the defining curve, more strongly in the region of short duration than elsewhere. Existence of damping implies that the tolerable spring distortion inferred from data plotted in the coordinates

of Fig. 1 is actually slightly less than that calculated from the formula

$$x_M = \frac{V_o T_n}{2\pi} .$$

This formula, however, can be safely used to compute strains for comparison with response spectral strains provided sprectra of the untested motions are calculated without damping.

E. Oscillating Input Motion

When the base motion is not one-sided but consists of oscillations in the direction of acceleration, the response of the simple linear elastic model can be substantially different from that discussed above, particularly if the frequency of the imposed oscillation is near the natural or resonant frequency of the model. Base motion consisting of one-half of a sinusoidal cycle at the resonant frequency is still one-sided and can therefore be treated in terms of the asymptotes (A_o, V_o), but one full cycle at the resonant frequency leads to peak strain that is twice as great as that stemming from the one half cycle of imposed sinusoidal motion. Two full cycles produce six times the peak strain of one half cycle at the same amplitude (Ref. 5). Thus, a continuing oscillation at or near the natural frequency can very quickly destroy a simple oscillator by introducing strain of an intolerable magnitude.

Two factors mitigate the severity of the response to continued forced oscillation however: frequency displacement and damping. At forcing frequencies below 1/2 or above 3/2 the resonant value the danger is much reduced; as the forcing frequency departs from this critical range the effect becomes more and more attributable to individual half cycles, i.e., a series of independent one-sided pulses (Ref. 5). Damping on the other hand, works most strongly within the critical frequency range. Damping amounting to 10% of critical can reduce the hazard from steady vibration at the resonant frequency below the level caused by two full cycles of the same frequency and amplitude applied in the absence of damping. (Analysis of the effect of damping can be

found in Ref. 4.) About 30% critical damping is needed to reduce the threat from prolonged oscillation near the resonant frequency to equal the threat to an undamped system when the forcing motion is a half-cycle of the same frequency and amplitude. Unfortunately, the exact degree of damping present in the important bio-mechanical systems is not so easily observed, but observations of human bodily reactions to steady vibrations suggest damping between 10% and 30% of critical is reasonable.

F. Sensitivity of Coupled Systems

A compound system consisting of two coupled subsystems may be more or less sensitive to a given base motion than either one of the components; however, the shape of the sensitivity diagram for the compound system is often not greatly different from that illustrated in Fig. 1. When natural frequencies f_1 and f_2 and masses m_1 and m_2 of the two separate components are near each other, coupling increases the sensitivity of both, as shown in Figs. 2 and 3 by the cases a, b, c, and d. These two figures present curves defining limiting distortion of both members of a tandem system when a rectangular acceleration pulse is applied to the base. In Fig. 2 which contains the sensitivity curve for the first oscillator the unit of time is the natural period f_1, of the first oscillator when alone; unit of speed is the instantaneous speed change necessary to produce the limiting strain in the first oscillator when alone. Similarly the units used in Fig. 3 refer to the second oscillator alone. (Mathematical derivation of these curves is carried out in Appendix B.)

In cases e and f, where $m_1/m_2 = 10$ and 20 and $f_1/f_2 = 1.43$, the first system is unaffected by the presence of the second but the second is strongly endangered by the sinusoidal response of the primary at a frequency near the secondary resonance. In all these cases the shapes of the sensitivity diagrams are similar to that represented in Fig. 1, and the empirical process of extracting values of V_o and T from observations may proceed as if only one harmonic system were involved, provided both of the limiting asymptotes, V_o and A_o, arise from a single subsystem.

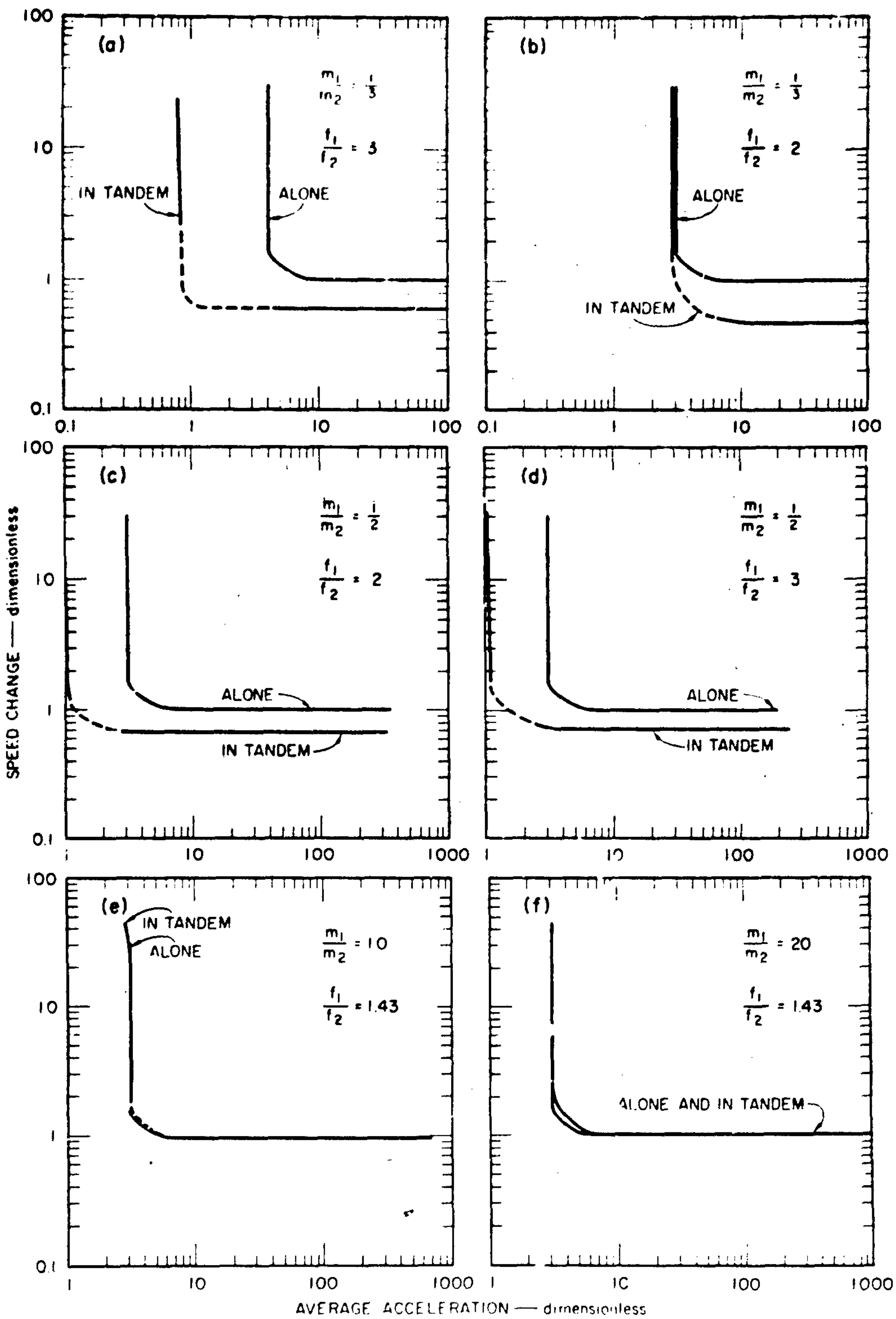

FIG. 2 SENSITIVITY CURVES FOR PRIMARY OSCILLATOR

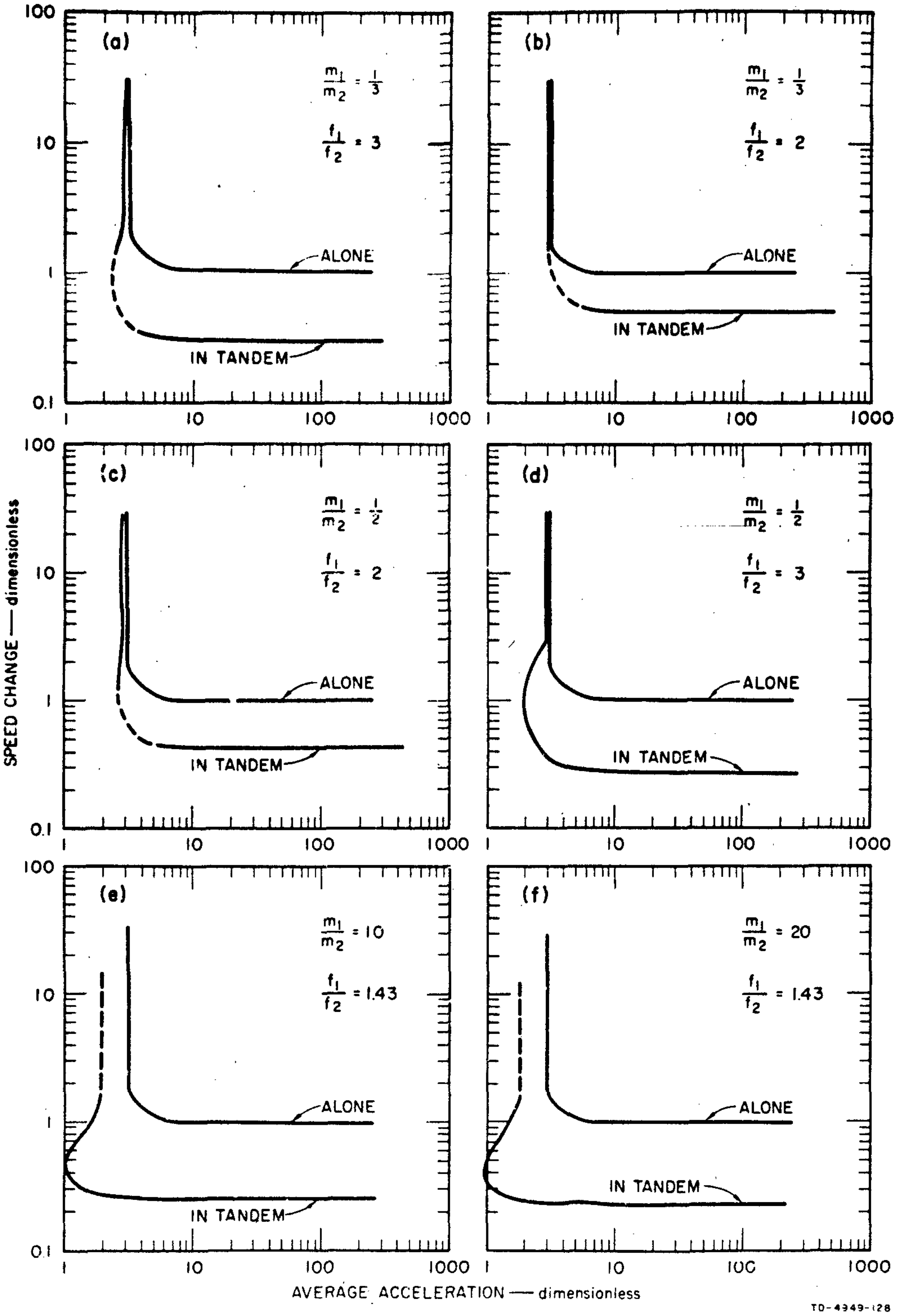

FIG. 3 SENSITIVITY CURVES FOR SECONDARY OSCILLATOR

When this is not the case and the two or more sensitivity diagrams are put into compatible units, overlapping may occur as suggested in Fig. 4, adapted from Kornhauser (Ref. 8). The dashed segment in Fig. 4 illustrates how a multicomponent system can safely be treated as a simple oscillator with an effective "frequency" somewhat different from any of the model frequencies of the complex system.

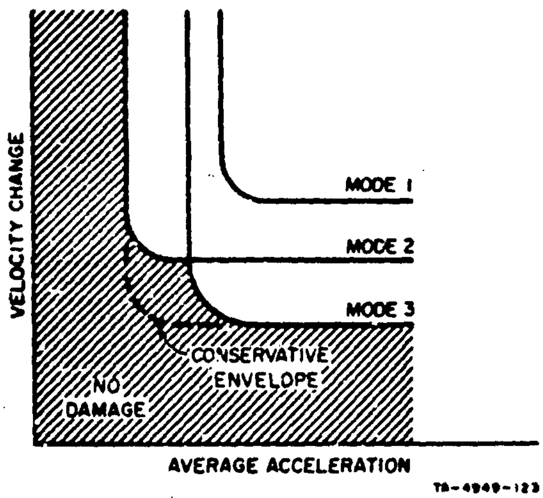

FIG. 4 SCHEMATIC DIAGRAM OF SENSITIVITY CURVES FOR SYSTEM WITH SEVERAL MODES OF FAILURE (After Kornhauser, Ref. 9)

To place curves in Fig. 3 in the same coordinates with those of Fig. 2, the relative tolerable distortion must be known, as well as parameters of the component subsystems. For example, if tolerable distortion in the primary is n times greater than that in the secondary and $f_1/f_2 = r$, then ordinates of Fig. 3 must be multiplied by 1/rn and abscissas by $1/r^2n$. Often f_1 and f_2 are near the observed modal frequencies; tolerable strains are more obscure quantities but some estimates will be attempted later for bodily subsystems.

Even when a single failure mode supplies both limiting asymptotes, A_o and V_o, the apparent natural frequency

$$f \approx \frac{1}{2T_o} = \frac{A_o}{2V_o}$$

may be shifted from that of the single system alone but because of the lack of precision in the relation above this shift is not usually observable.

Figures 5 and 6 are sensitivity diagrams for the cases $m_1/m_2 = 10$ and 20 and $f_1/f_2 = 1/5$. The primary subsystem, which is itself hardly affected by the presence of the secondary, acts over much of the range of pulse duration as an isolator for the secondary greatly reducing its sensitivity. However, protection against pulses lasting times nearly equal to the natural period of the secondary is not so effective as otherwise; and the diagram has an unusual shape. The wrong inferences could be drawn by attempting to extract the values V_o and T_n from observational data in the usual way.

All the foregoing cases treated in Figs. 2, 3, 5 and 6, were chosen because of their possible pertinency to the human body, as will be seen later.

G. Transition to the Shock Mode

It is not to be understood from consideration of Fig. 1 that acceleration undergone by a part or all of the structure may be unboundedly large as long as duration is reduced proportionately. Very brief acceleration pulses of great magnitude become shock fronts and affect the structural elements on a microscopic scale in a way to be considered later. For understanding of this failure mode, the simple oscillator model is inappropriate.

H. Response Spectra

As used in this report a response spectrum of a given motion specifies the maximum distortions of simple undamped harmonic oscillators of all frequencies whose bases move in the prescribed way. Presentation is through a plot of the product of circular frequency and maximum distortion, i.e., ωx_M, against frequency, ω.

Were the response spectrum of a motion and the sensitivity diagram of an oscillator both known, the tolerance of the harmonic system for the motion can be easily judged: the system is endangered if the value of the spectrum ordinate, ωx_M, at the natural oscillator frequency, $(2\pi/T_n)$, exceeds V_o.

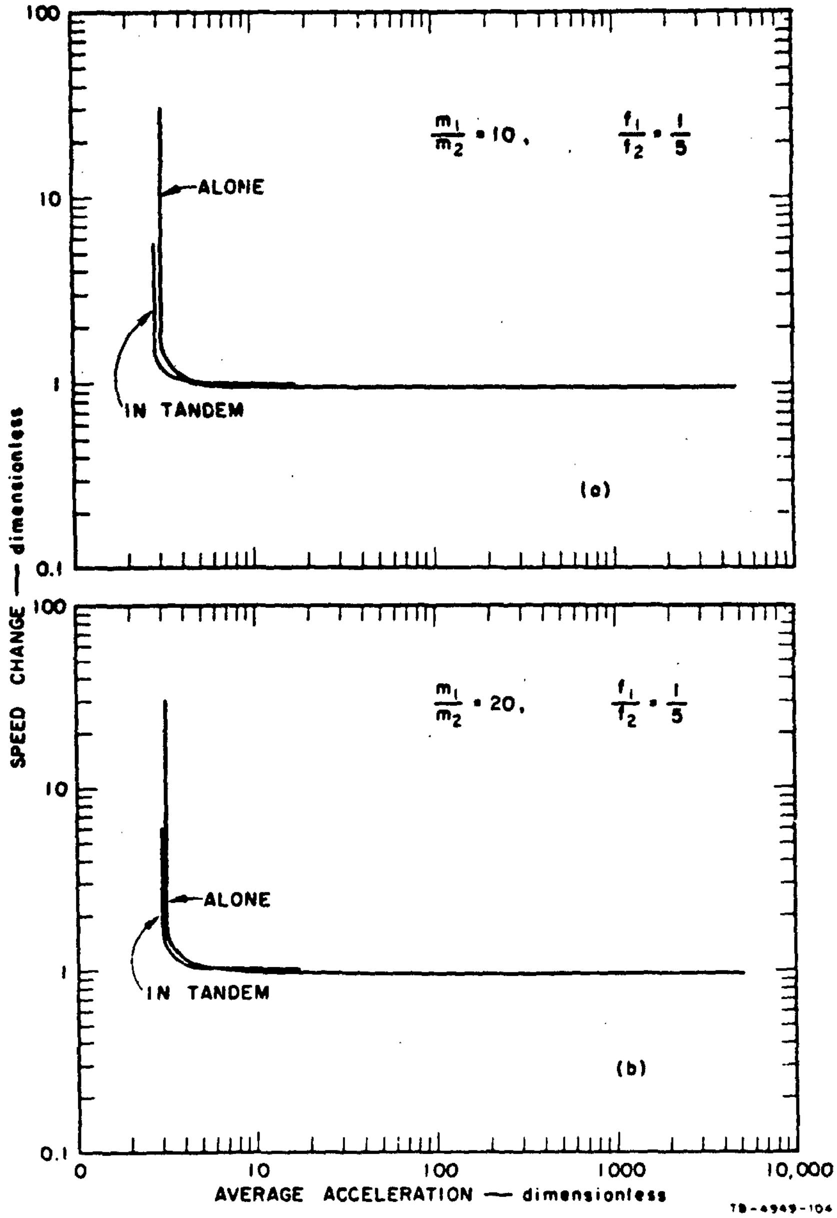

FIG. 5 SENSITIVITY CURVES FOR PRIMARY OSCILLATOR

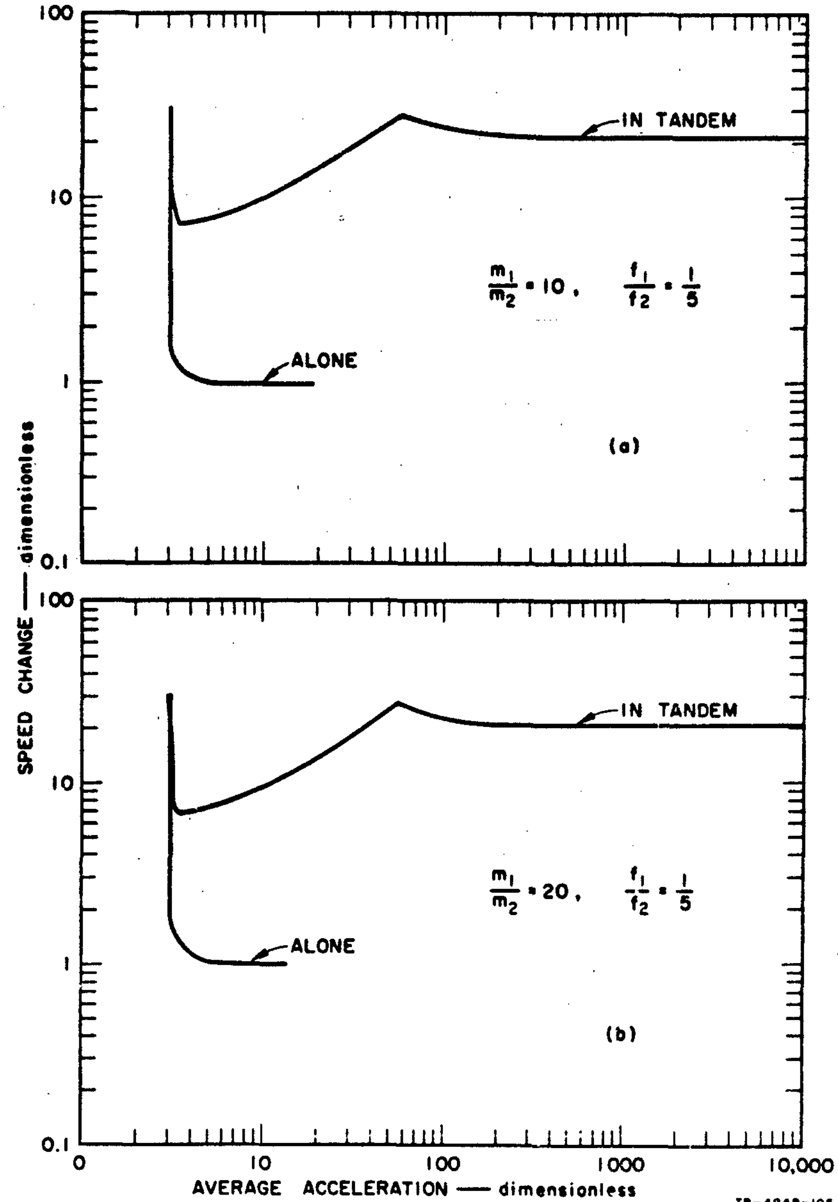

FIG. 6 SENSITIVITY CURVES FOR SECONDARY OSCILLATOR

The spectrum of a step in speed (acceleration delta function) is a horizontal line at an ordinate which, in the absence of damping, equals the value of the step. The reduction in the ordinate due to damping is given in Table I above.

Any system of n-coupled linear oscillators can be treated mathematically as n-coupled oscillators by transformation to normal modes (Ref. 35); however the forcing function is in effect transformed also. It appears to be an assumption that, if a n-degree of freedom linear system with n-normal mode frequencies is assigned n values of V_o, comparison of the V_o's with n response spectra ordinates at the n frequencies will be meaningful.

V THE MOTION ENVIRONMENT

Instruments placed in the ground in the neighborhood of nuclear explosions on the surface and in the atmosphere have provided a substantial body of knowledge of the reaction of the earth to the intense pressures produced in the crater by the explosion and over the surface by the expanding airblast. These data pertain to many different weapon yields and heights of burst but to only two environments, the Nevada Test Site and the Eniwetok Proving Ground. The instruments used have been generally of two kinds, (a) accelerometers, which provide a history of vertical and horizontal ground acceleration during the passage of the shock through the observation station, and (b) reed gages. The latter are essentially single degree-of-freedom linear oscillators which record maximum deflection reached during or shortly after the motion of the ground where they are attached. Ordinarily, several reed gages built to oscillate at different frequencies are put out at one station. Since these oscillators are small, the motions of their bases are the free-field ground motions at their locations.

Response spectra determined by these two ways will be used as follows: (1) ordinates at given frequencies will be looked upon as indexes of maximum threat offered to oscillators of these frequencies or to compound systems with these modal frequencies by the complex motion whose response spectrum is being used, and (2) any complex system that is unharmed by a complex motion with a specified spectrum will also be regarded as unharmable by any other motion whose spectrum falls within the specified spectrum. There are serious reservations about the validity of the second of these two propositions as applied to real systems (Ref. 36) but better practical theories have not been found.

A. Response Spectrum

By simple computation, an accelerogram taken at an observation station can be made to yield the responses of any number of reed gages at that

station,* and it is in terms of the response of these single degree-of-freedom oscillators that earth motion data from weapons tests have been summarized. Often, the summary has the form of a "response spectrum," as shown in Fig. 7, taken from Ref. 37. Here the logarithm of the product of maximum oscillator excursion x_M (relative to its base) and circular frequency ω appears along the ordinate and the logarithm of the circular frequency along the abscissa.

In these coordinates lines of slope +1 are loci of constant maximum displacement from equilibrium, and lines of slope -1 are loci of constant peak acceleration for the oscillating masses in the single degree-of-freedom oscillators. Several of these loci have been marked in Fig. 7 with the corresponding values of constant peak displacement and acceleration.

The data in Fig. 7 are based on four histories of vertical acceleration 5 ft below the Nevada surface at ranges from ground zero where peak airblast overpressures were from Tumbler 1 (airburst) between 6.7 and 10.8 psi. The first integrals of these records are reproduced in Fig. 8 (from Ref. 1) and make up the vertical speed histories from the four stations. This particular series of records contains several different combinations of the two constituent wave types often seen in ground motion observations from atmospheric bursts. The sharp downward thrust (beginning at time labelled AB in Fig. 8 and ending some 50 msec later) stems from the airslap on the surface directly above the recording gage. The remainder of the wave--a rolling undulation--is thought to represent the result of an earlier airslap after transmission through the ground to the gage--probably also after reflection from a hard, deep layer. The wave at Station 4 is mainly a direct airslap while farther from ground zero at Station 7 the direct airslap component has become weak (although the maximum speed and acceleration are still due to direct

* The degree of damping must be assumed and it is generally taken at 0.5% of critical (Ref. 1). This is a realistic estimate of the actual damping present in a reed gage. The question of the amount of damping to assume for a system subject to motion can be important and a reasonable, conservative procedure is to assume none.

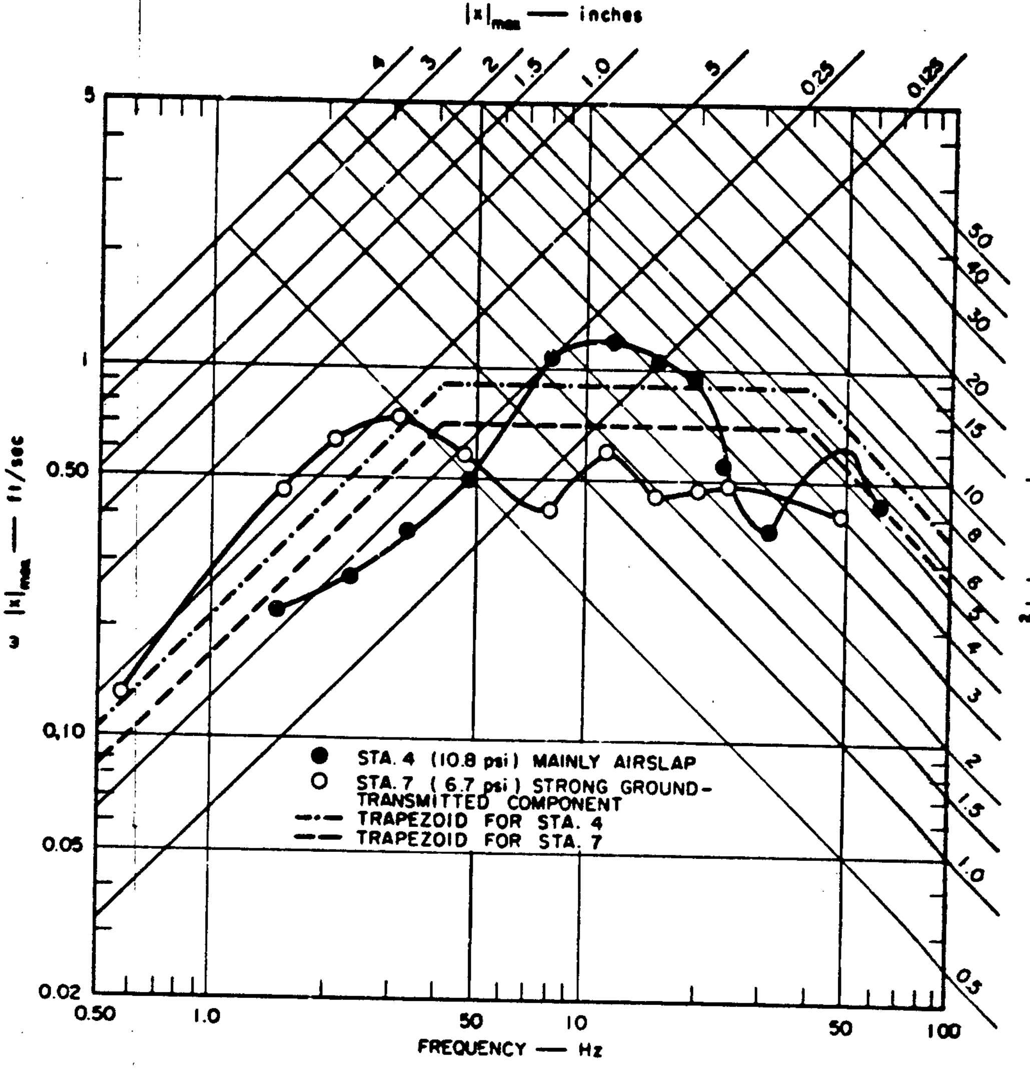

FIG. 7 COMPUTED SPEED RESPONSE SPECTRUM FROM GROUND MOTION HISTORIES, SHOT TUMBLER 1 (5-ft depth, 0.5% critical damping) COMPARED WITH RESULTS OF TRAPEZOIDAL RULE (Airburst)

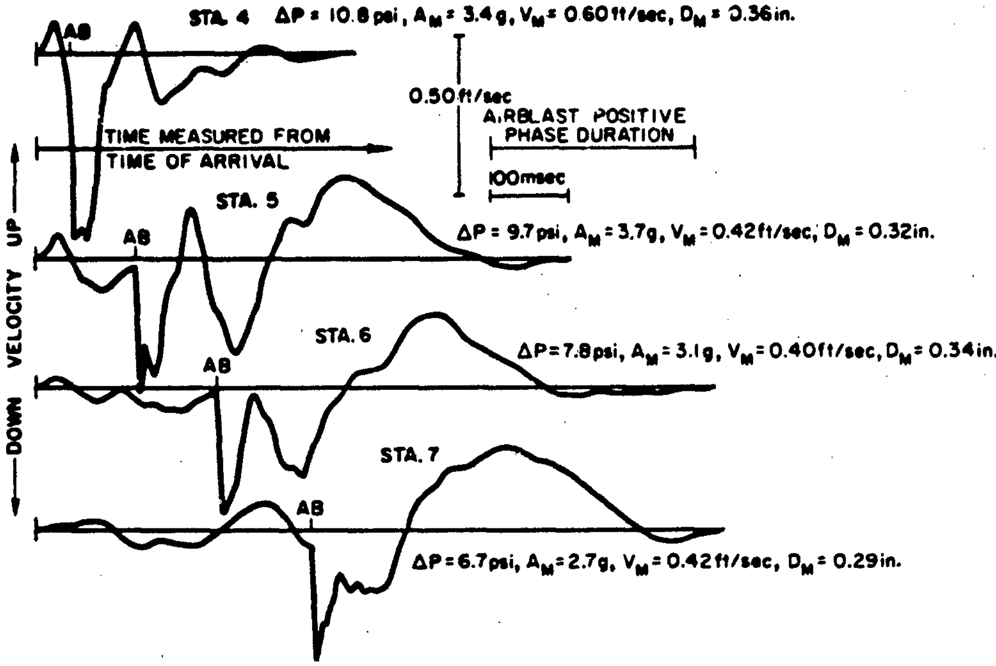

FIG. 8 VERTICAL GROUND SPEED HISTORIES, SHOT TUMBLER 1, 5-ft DEPTH
(After Sauer, Ref. 1)

airslap). This situation reflects the fact that airblast wave speed falls rapidly with overpressure in the range from 0.1 to 1 atmosphere but ground wave speed at corresponding pressures remains fairly constant.

Also listed in Fig. 8 alongside each waveform are peak airblast overpressures (ΔP), maximum absolute accelerations (A_M), highest absolute speeds (V_M), and peak displacements (D_M) seen at the corresponding station.

B. Trapezoidal Rule

Empirically, it has been found that measurements can often be summarized still more simply than by a computed or observed response spectrum. In Fig. 7 a dashed line has been drawn along a part of the locus of constant peak displacement corresponding to the value of observed maximum displacement of the ground at Tumbler 1 Station 4; another dashed line appears on the ordinate equal to 1.5 times the observed maximum ground

speed at Station 4, and there is a final section of the dashed line along the locus of constant maximum acceleration equal to twice the observed peak ground acceleration at the same station. The dashed line very nearly envelops the response spectrum. When the corresponding bounds are drawn from the data from Station 7 (where the ground transmitted motion is more important) departures from the simplified envelope at low frequencies are evident. This suggests that the simple trapezoidal spectrum shape may not be valid at all likely detonation sites, particularly at those containing hard soils where ground transmitted motion is more important than at the Nevada Test Site.

C. Parameter Study

Results of a theoretical parameter study suggest the doubts may be justified (Ref. 32). For this study observed vertical ground speed wave shapes were idealized and simplified into two components [in the manner of Sauer (Ref. 1)], one associated with the force of the airblast on the surface above the gage and another tentatively identified with reflections and refractions from geologic strata below the gage of earlier airblast forces on the surface closer to the burst point. These idealized components were then added together in 29 different ways (that is, with different relative amplitudes, durations, and phasings) and nondimensional undamped response spectra computed for them and for five actual waveforms, making a total of 34 different spectra. Among the twenty-nine synthetic waveforms two were pure Type I and Type II and twenty-seven were combinations. In the combinations the relative amplitudes of the Types I and II waves were 2:1, 1:1 or 1:2; relative durations were permuted through the same three ratios. Three different phasing arrangements were also used; that is, the two shapes were centered at the same point in time, the shorter wave was centered at the start of the longer or the shorter was centered at the end of the longer. The result is contained in Fig. 9 (adapted from Ref. 32) where the ordinates are those of Fig. 7 divided by the maximum ground speed found in the whole wave and the abscissas are products of oscillator frequencies and wave durations, T. The higher curve in Fig. 9 traces the envelope of all of the 34 individual response spectra and the dashed curve is an average. The computed response data

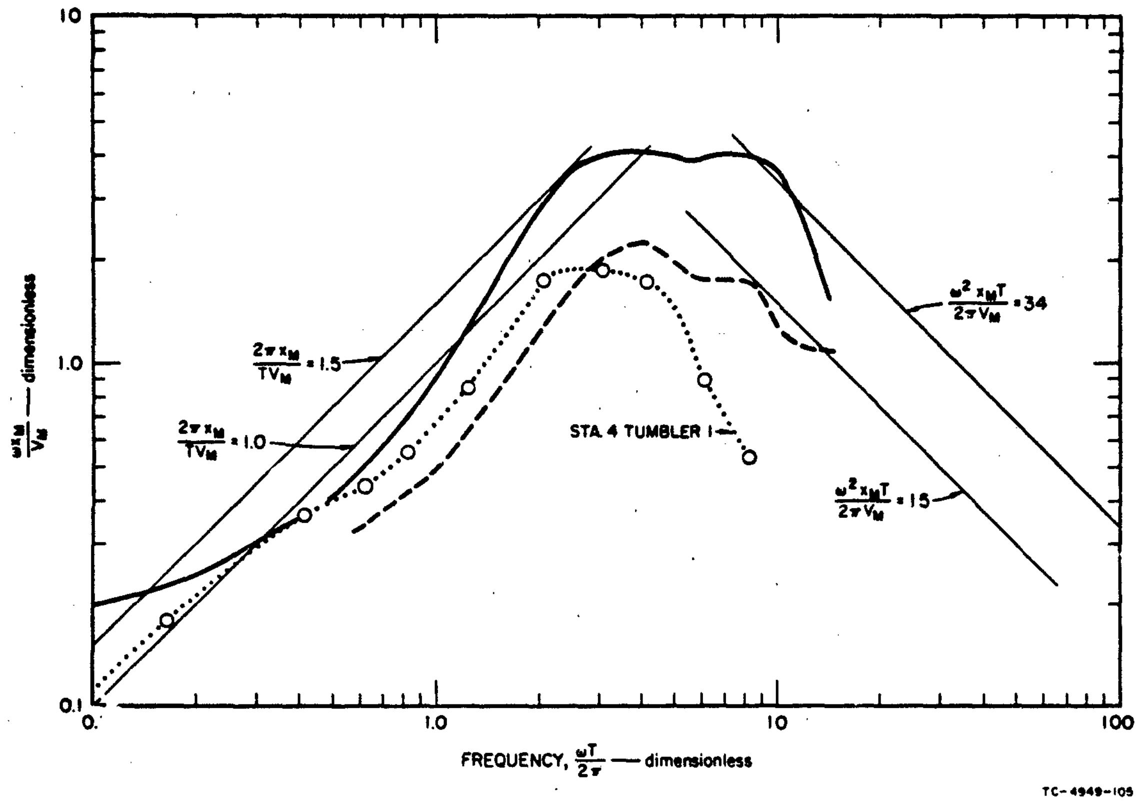

FIG. 9 UNDAMPED SPEED RESPONSE SPECTRUM RESULTING FROM PARAMETER STUDY

used in Fig. 7 from Station 4 at Tumbler 1 has been converted to nondimensional scales and appear in Fig. 9 connected by a dotted curve. The most striking difference between the envelope of the responses to the synthetic waveforms and the response spectrum from Tumbler 1 is the generally greater ordinates in the envelope, where the peak ordinates are not in the range 1.5 to 2.0 as would be expected from Fig. 7 but reach to 4.0. Thus the peak strains in simple oscillators exposed to motion elsewhere than Nevada may be greater than expected from analysis of Nevada data. Also the low frequency portion of the envelope is not even approximately straight and tends to lie outside the region expected from observations in the region where direct airslap predominates; this indicates the likelihood of greater maximum strains ($2\pi x_m/TV_M$ in the nondimensional units of Fig. 9) at certain frequencies than are found in Nevada Test Site spectra. The peak accelerations in Fig. 9 ($\omega^2 x_m T/2\pi V_M$) given by the artificial waveforms are not thought to be realistic.

Because the factor of 4 is based on an envelope of a wide array of wave types, some of which may be unrealistic, and because no damping was assumed in the calculation of the response spectra of Fig. 9, its use is considered highly conserv-tive. It should be noted too that effective application of Fig. 9 requires some estimate of both peak vertical ground speed V_M and wave duration T at a point.

A simple oscillator exposed to the earth motion summarized by the envelope will suffer maximum contraction or stretching less than or equal to that given by the ordinate of the intersection of the envelope and the abscissa corresponding to the oscillator frequency. In view of the simple model described in Section IV-B, a response spectrum envelope may provide what is needed to assess the likelihood of damage to a person or to equipment exposed to free-field earth motion. Even when a response spectrum is not available, the motion environment of objects can be estimated by the help of the trapezoidal rule from only three values: peak displacement, maximum speed, and acceleration of the base on which

objects are mounted.* If the ground motion is expected to be predominately due to direct airslap, the trapezoid will consist of the lines D_M, $1.5V_M$ (or possibly $2V_M$) and $2A_M$ as suggested in Fig. 7. If the character of the motion however is partly undulatory or if the character is unknown, then the more conservative upper boundary $4V_M$ suggested by Fig. 9 may have to be used; location of the other boundaries then requires an estimate of pulse duration T. The high frequency boundary of the trapezoid found by this procedure will probably be very conservative.

D. The Peak Parameters of Motion

The next step in the description of the free-field motion is assignment of values to the parameters D_M, V_M, A_M, and T. When the motion comes from direct airslap and the peak airblast overpressure is known, very good estimates of all parameters but A_M can be made. Airblast overpressure profiles or histories at fixed surface points are available in the literature (e.g., Ref. 38) but even for peak overpressures ΔP_M as high as 50 psi good estimates of ground motion can be made by assuming a triangular waveshape, i.e.,

$$P - P_o = \Delta P \left(1 - \frac{t}{T_1}\right) \quad \text{for } o \le t \le T_1$$

where $P - P_o$ is the instantaneous overpressure and T_1 the time duration of positive overpressure. In this case pulse duration in the ground T is the same as duration of positive overpressure in the airblast T_1. (Since duration of airblast positive phase will always be greater than or equal to duration of the resulting downward phase of earth motion, use of the equality will be conservative.) Thus if U equals (constant) shock speed in soil of density ρ_o:

* Merritt and Newmark (Ref. 7) assert that there are very rare cases when the trapezoidal rule predicts oscillator peak displacement which is too low by a factor of 2 but in view of the much larger uncertainties in the forecast of the motion environment they do not consider such a possible error important.

$$\text{Particle speed } V = \frac{P - P_o}{\rho_o U} \quad \text{or} \quad V_M = \frac{\Delta P}{\rho_o U}$$

$$\text{Displacement } D = \int_o^{T_1} V\, dt \quad \text{or} \quad D_M = \frac{1}{2} \frac{\Delta P}{\rho_o U} T_1 \quad .$$

Values of T_1 are also widely available in the literature of nuclear airblast phenomena (e.g., Refs. 38 and 39). The quantity U is more difficult to find. For any hard rock 50 psi is well below the elastic yield point and U is equivalent to sound speed c in the same material. In alluvium or other easily crushable material U may be significantly lower than c and may in addition depend on peak overpressure. Unfortunately propagation speeds of waves carrying peak overpressures in the range 1 to 100 psi have not been widely observed. Reference 1 suggests

$$U \cong \frac{3}{4} c$$

for Nevada Test Site playa in this pressure range.

Reflections moving upward from layers below the surface can interrupt the development of the direct airblast induced ground wave. Reflections are not likely to influence V_M or A_M but can be expected to reduce D_M.

Estimation of A_M is also empirical. When airblast is superseismic the rise time of the airblast wave determines an upper limit but any soil so rapidly diffuses and/or disperses the input wave that such a limit is not useful.* Fortunately peak acceleration is important only at the high frequency end of the response spectrum where many objects found in shelters have good built-in filters. Experience from nuclear tests on and over Nevada Test Site playa suggests a direct proportionality

* Reference 7 contains a qualitative discussion of the influence of geometrical dispersion in broadening the wave front at depth. The method used to determine below ground wave shape neglects inertial as well as dissipative effects and the indicated values of A_M cannot be taken as numerically correct.

between peak airblast overpressure and the resulting peak downward acceleration in which the constant of proportionality varies between 0.20 and 0.60 g/psi* (Refs. 1, 37), the corresponding constant does not appear to have been calculated for Eniwetok soils because the highest value of acceleration there could not usually be unambiguously associated with the direct airslap.

Were the value of A_M due to direct airslap determined by the rise time of the input surface pressure alone, then it would clearly decline with increasing wave speed in the soil. (Rise time of the wave would be unchanged but peak speed change is inversely proportional to product of density and wave speed.) However, one measurement at Flat Top I,† 18 inches below the surface of a limestone outcropping where the peak airblast overpressure is about 90 psi suggests that this is not true and that, as indicated above, dissipative and/or dispersive factors may be controlling at these overpressure levels. This one accelerometer reading points to a relationship close to 1.0 g/psi for limestone, in which the seismic speed is approximately 19,000 ft/sec as contrasted to the speed of 1000 ft/sec in Nevada playa. Limestone, even in the absence of geometrical divergence, shows far less wavefront broadening with wave travel than does a highly incompetent material like playa. Thus, while assumptions of elastic behavior and the simple conservation laws of mechanics give good guidance in estimating the influence of direct airslap on the quantities D_M, V_M, and T_1, the value of A_M must in general be sought elsewhere.

For the ground transmitted portion of the motion in the free field around a shelter, the estimation of the motion parameters is much more difficult than the procedure outlined above for the airslap portion and it is especially here that warnings must be made about the danger of applying Nevada and Eniwetok experience to all and any soils. Both the

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* When the airblast waveform is not ideal, i.e., there is a precursor running through disturbed air near the surface, this constant will be much reduced.

† A surface explosion of 20 tons of TNT on Nevada limestone. Ref. 40.

kinds of soils and their layering are important factors and wide enough experience does not exist for confident quantitative forecast of their influence (even though, as noted earlier, the wave shape may well be a general feature of this motion). Also it is not certain that the degree and rate of coupling of explosive energy to the soil in the neighborhood of ground zero is not of some importance at ranges where peak overpressure is 50 psi or less. Presumably, a burst at a height such that the highest air overpressure on the ground is, for example, 50 psi would introduce the same total momentum into the ground as a surface burst but perhaps the momentum that is introduced by a burst at altitude is spread more widely over the ground so that its effect at a given point is less intense than that of the same momentum put into the ground over a relatively small area around ground zero.

The pertinent nuclear experience of ground transmitted motion has been summarized in Refs. 1 and 37 and will be repeated here for convenience of the reader. The vertical speed profile has been quite generally (both at Eniwetok and Nevada) approximately two cycles of movement, first upward, reaching a peak on the second upward swing and then falling rapidly in amplitude (Fig. 10). Duration of the first cycle at a certain range is given in terms of the difference, ΔGR, in feet between the distance from ground zero at which outrunning first takes place and the range considered, as follows:

$$T_2 \text{ (msec)} = 100 + \frac{\Delta GR}{4} \text{ (ft)} \qquad . \qquad (1)$$

At points where ΔGR is less than zero the ground-transmitted portion of the ground motion has been negligible contrasted to the direct airslap induced component. Total wave duration is given as $2.5T_2$.

Peak acceleration A_M in g's in the transmitted component has been expressed as follows:

$$A_M = 10^{10} \left(\frac{R}{W^{1/3}}\right)^{-3.5} \quad \begin{matrix} +200\% \\ -70\% \end{matrix}$$

$$150 \le \left(\frac{R}{W^{1/3}}\right) \le 800$$

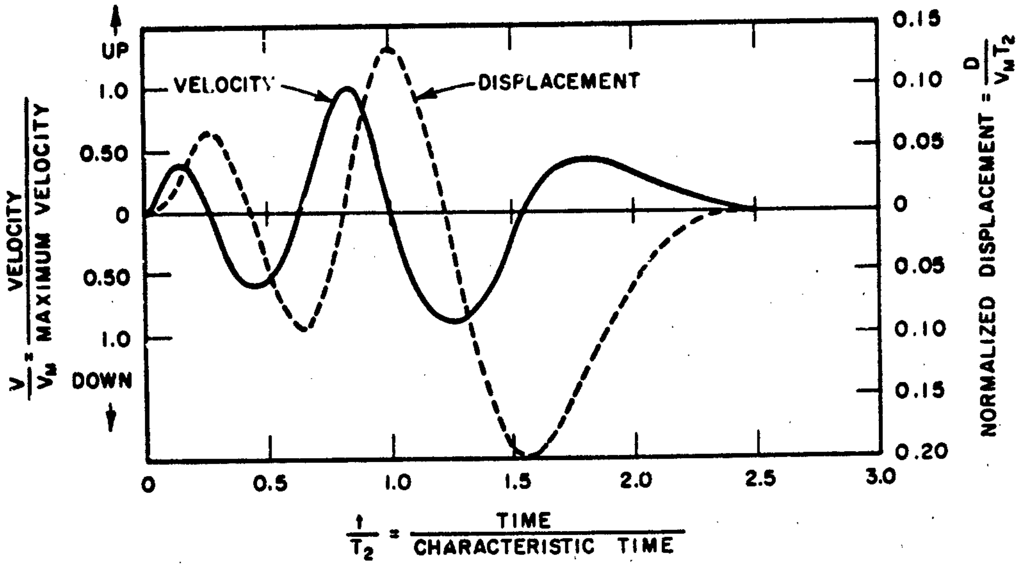

FIG. 10 IDEALIZED GROUND-TRANSMITTED WAVE (After Sauer, Ref. 1)

and

$$A_M = 5 \times 10^5 \left(\frac{R}{W^{1/3}}\right)^{-2} \qquad \begin{matrix} +200\% \\ -70\% \end{matrix}$$

$$800 \le \left(\frac{R}{W^{1/3}}\right) \le 3000 \qquad (2)$$

where R is range in feet from ground zero and W is weapon yield in kilotons. This correlation is based on observations at eight surface (or very near surface) bursts at Eniwetok and one surface burst at Nevada and its use for airbursts is conservative.

It should be noted that peak acceleration in the ground transmitted component will normally be much less than peak acceleration in any direct airslap component which may be clearly discernible. At least that has been the experience so far.

Maximum vertical ground speed V_M in ft/sec in the transmitted wave is given as

$$|V_M| = 5 \times 10^5 \left(\frac{R}{W^{1/3}}\right)^{-2} \qquad \begin{matrix}+60\% \\ -40\%\end{matrix} \tag{3}$$

and the data is entirely from near surface bursts at Eniwetok. This is a "proven" maximum for the rolling component and at the 50 psi range amounts to 2 $\begin{matrix}+60\% \\ -40\%\end{matrix}$ ft/sec. Apparently, the vertical ground speed stemming from direct airslap was larger than that due to the transmitted component at all observing stations in Nevada so that the relationship for airslap motion at large ranges can be used as an upper bound on ground-transmitted amplitude, i.e.,

$$|V_M| \leq 20 \times 10^5 \left(\frac{R}{W^{1/3}}\right)^{-2} \qquad \begin{matrix}+100\% \\ -50\%\end{matrix} \tag{4}$$

for the transmitted wave in Nevada soil.*

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* Sauer (Ref. 1) notes a relation between particle speeds at Eniwetok and NTS and the inverse ratio of seismic speeds at the same two sites. His method of motion prediction (given by Brode, p. 53, Ref. 38) makes peak particle speed V_M in the ground-transmitted wave inversely proportional to soil density and seismic speed, i.e.,

$$V_M = 4 \times 10^3 \frac{1}{Sc} \left(\frac{R}{W^{1/3}}\right)^{-2}$$

where S = specific gravity, c = seismic speed (ft/sec), W = yield in kt, R = range from ground zero in feet. Although it is plausible that more rigid earth materials will show less particle motion than less rigid, such a relation can hardly be considered well established on the basis of experiments in two media; but, even so, no writer attempts to treat the case of a strongly layered medium. What is the near surface motion, for instance, in a light soil underlain by a hard rock? Equation 4 above based on NTS data but used at overpressures where airslap rather than ground transmission predominates at NTS serves as an upper bound for the magnitude of the rolling motion anywhere.

Murphy in forecasting peak parameters in the ground-transmitted wave under hypothetical attacks in five specified locations accounts for layering by an interpolation procedure whereby near-surface motion after outrunning is influenced by the different kinds of soils down to a depth of 200 ft (Ref. 87).

At the 50 psi range, i.e., where $R/(W^{1/3}) = 500$ ft, this equation yields

$$|V_M| = 8 \begin{smallmatrix}+100\% \\ -50\%\end{smallmatrix} \text{ ft/sec} \quad . \tag{5}$$

The waveforms on which Eq. 5 above is based often show a maximum ground speed in the rolling component equal to more than one-half the peak in the whole waveform. For this reason and in view of the large uncertainty limits above, $V_M = 8$ ft/sec is a reasonable peak "expected" or "extrapolated" vertical speed in the ground-transmitted wave.

The best direct measurement of maximum vertical displacement D_M in the purely transmitted wave stem from five surface shots at Eniwetok and will be summarized by the expression

$$D_M = 7.5 \times 10^3 R^{-2} W \qquad \begin{smallmatrix}+100\% \\ -50\%\end{smallmatrix}$$

where both D and R are in feet (Ref. 8).

Sauer (Ref. 1) integrates his idealized speed profile due to transmission to achieve zero final displacement and peak transient displacement downward

$$D_M = \frac{1}{5} V_M T_2$$

and peak transient displacement upward

$$D_M = 0.13 V_M T_2 \quad .$$

This is formally inconsistent with the simple relation above giving D_M as a function of R since V_M is inversely proportional to $W^{2/3}$ but T_2 does not depend on $W^{1/3}$ in any simple way. In fact, predictions of peak displacement made in these two ways can differ by several orders of magnitude. The simple inverse square law generally yields larger values of D_M.

The value of P corresponding to any given peak overpressure will be found in Ref. 39. A 1-kt surface burst, for example, produces a peak overpressure equal to 50 psi at approximately 500 feet from ground zero.

Horizontal or radial surface motion is commonly given as a fraction of the vertical. According to Ref. 1, which is the source of all statements in this paragraph, Nevada data show that peak acceleration A_M in the radial direction is nearly always due to direct airslap and amounts to 0.2 and 0.5 times A_M in the vertical direction. At Eniwetok, however, the ground transmitted motion produces peak horizontal A_M equal to about 0.5 times the vertical. Peak horizontal speed V_M due to direct airslap in Nevada is outward and varies between 1/10 and 1/4 the value of V_M for vertical airslap motion. The waveform is apparently not nearly so purely one-sided as its vertical counterpart; the following inward peak is only 30 to 50% lower than the main jump in outward speed. Eniwetok experience is almost wholly with ground transmitted motion and shows peak horizontal speeds roughly comparable to vertical. No information on horizontal speed history is given in Ref. 1 but the field data from Eniwetok (Ref. 41) at overpressure below 100 psi make it appear reasonable to attribute the same waveform to the horizontal speed as to the vertical (see gage records 13H10 and 13H100, Ref. 41). Horizontal speed profiles due to direct airslap seem to be similar to vertical except that the horizontal rebound (inward motion) is a much greater proportion of the initial outward movement than the vertical rebound (upward) is of the first downward thrust (Ref. 42).

The only known comparison of peak D_M for the horizontal and vertical components of motion has been made by Sauer (Ref. 1) from which it appears that when the motion is ground transmitted both components are roughly the same.

Depth attenuates the vertical component of direct airslap induced motion rather markedly. Measurements underlying the statements made in the foregoing paragraphs concerning this component come from scaled depths of 5 $ft/kt^{1/3}$ or less. At scaled depths of 30 $ft/kt^{1/3}$ peak particle speed V_M due to airslap can, for example, be one-half its

surface value.* Variation of maximum airslap displacement with depth is not so well understood nor has it been so accurately observed. (See Ref. 43 for a complicated calculational procedure of doubtful validity.) Moreover, for megaton weapons 30 ft/kt$^{1/3}$ corresponds to depths of burial over 300 ft, which is generally an unlikely depth. Therefore, in the interest of conservatism, attenuation of vertical airslap motion with depth below 5 ft will be overlooked in this report. In any case, horizontal airslap motion changes very slowly with depth. Variation with depth of the transmitted portion of the ground motion depends strongly on the geologic structure of the shelter environment. Observations have usually shown peak values of the parameters falling with depth but not always; rather, the horizontal parameters have sometimes increased with depth at Eniwetok. In any case the rate of decline, when it does exist, is much slower than for the airslap motion and will be ignored.

We must close this portion of the discussion of the motion environment with another warning of the inadequacy of present knowledge of near surface ground motion brought about by nuclear explosions near the surface. In particular, the influence on the transmitted component of all the wide range of likely stratification patterns has not been explored theoretically or experimentally. Also, very high seismic speeds may change the character of near surface motion drastically. Hitherto, for example, the Rayleigh wave--a violent disturbance occurring only near the surface--has not appeared in experiments because of its slow speed. The authors of Ref. 7 suggest there may be a chance to feed energy steadily from an advancing airblast into such a disturbance--if the Rayleigh speed (generally 1/3 to 1/2 the compressional wave speed) were larger than it is at test sites used up to now.

The foregoing information can be summarized in terms of one or more likely shock spectra, in a way that will result in conservative estimates

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* Rate of attenuation appears to be much greater than this at Eniwetok, perhaps due to the very high water table (which is found 2 to 5 ft below the surface). In fact, by a depth of approximately 2.5 ft/kt$^{1/3}$ the direct airslap contribution to vertical ground speed is no longer distinguishable from the transmitted portion.

of the effects of motion on shelter contents at any likely location. Obviously the first restriction is to consider only surface or near surface bursts; this is a likely form of attack, and ground motion at a point resulting from airbursts will always be less than the motion at the same point due to a surface burst having the same ground zero. A second restriction in the interests of conservatism is to neglect damping; spectra appearing in this section will, unlike that of Fig. 7, be calculated without considering damping. (The importance of damping depends upon the shape of the acceleration pulse. For an oscillatory pulse the effect is stronger than for a one-sided pulse. Compare values of V_O in Table I with the ordinates in Fig. 11. For a one-sided pulse the importance of 5% critical damping is negligible.) The airblast wave speed at 50-psi overpressure is approximately 2400 ft/sec; hence shelters at the 50-psi range in soils in which all significant seismic speeds are much less than 2400 ft/sec, such as those found at the Nevada Test Site, will be exposed to pure airslap motion. Soil materials found in the upper layers at Nevada Test Site are very likely extreme in having very low shock impedance (product of original density and shock speed). Nevada Test Site seismic speed of 1000 ft/sec (or 0.30 mm/μsec) equals nearly the least of those listed by Press (Ref. 44) for all kinds of soils; its specific gravity, 2.0, exceeds only that of loosely compacted dry sand. Thus the 50-psi overpressure airslap upon this material will come close to defining the highest values of V_M and D_M needed in the construction of a free field, purely airslap spectrum. In Fig. 12 the trapezoidal rule has been applied to draw two curves from the values: V_M = 2.5 ft/sec, and A_M = 30 g to represent the response of Nevada Test Site playa and V_M = 1.7 ft/sec and A_M = 150 g that of hard rock. There are families of left boundaries to each spectrum, each boundary corresponding to one of several realistic yields, 100 kt, 1 mt and 10 mt.

Figure 9 introduces the possibility that if there is a strong ground-transmitted wave, response spectral ordinates may be as high as four times maximum ground speed. If the reasonable "extrapolation" is accepted for the 50-psi range, i.e.,

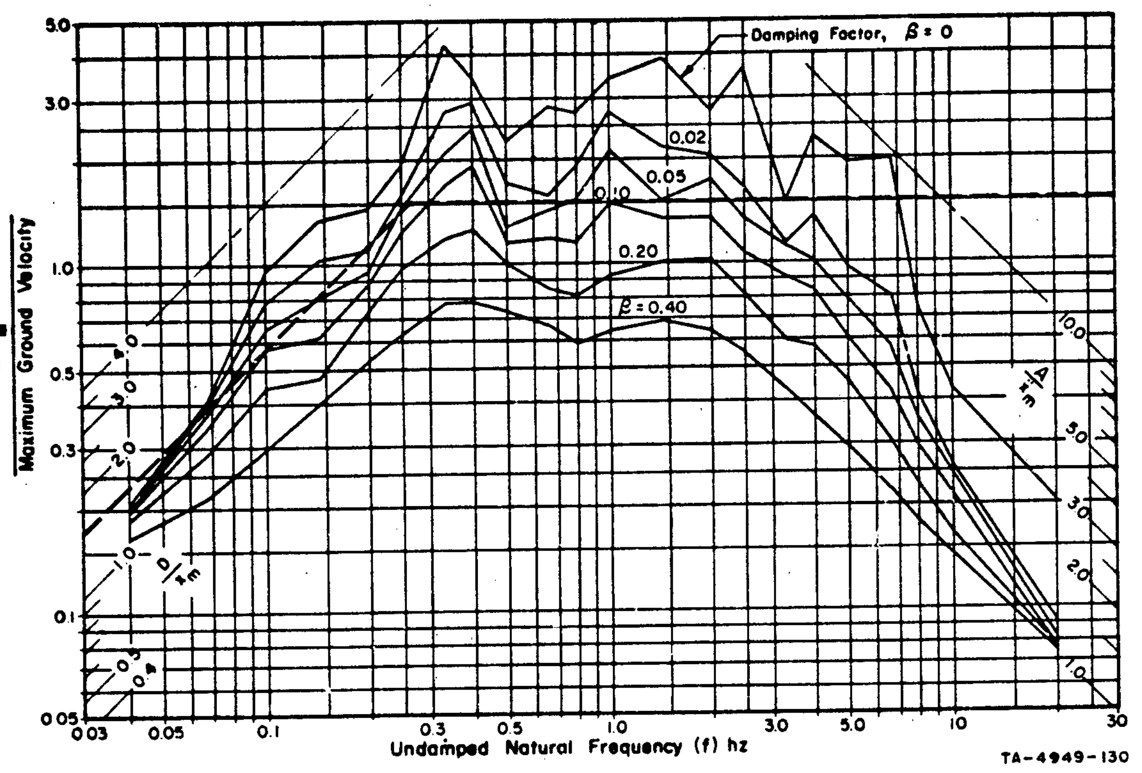


FIG. 11 EFFECT OF DAMPING ON SPECTRUM OF OSCILLATORY MOTION (from Ref. 90)

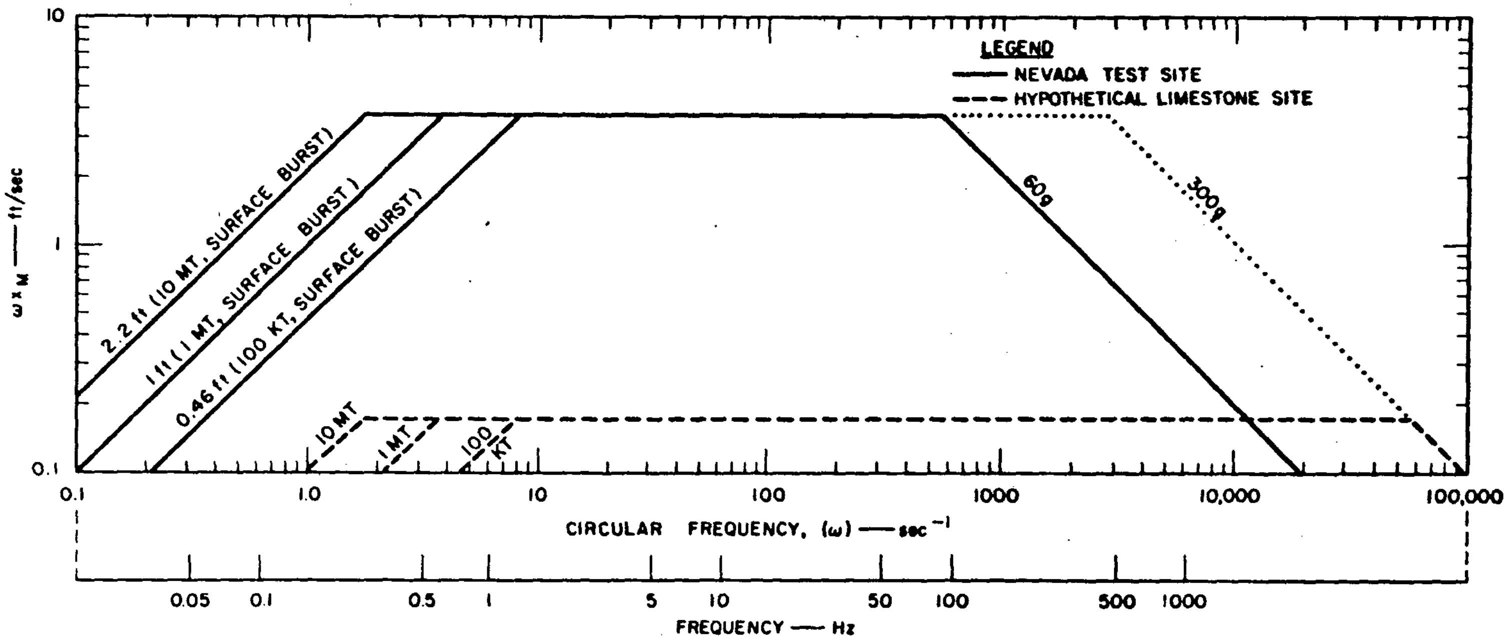


FIG. 12 VERTICAL RESPONSE SPECTRAL ENVELOPES FROM A 50-psi AIRSLAP GROUND MOTION

$$V_M = 8 \text{ ft/sec},$$

ordinates as large as 32 ft/sec are implied; the "proven" maximum ordinate is 8 ft/sec. The region of the frequency axis covered by ordinates of this magnitude depends critically on geology. If the seismic wave does indeed outrun airblast between ground zero and the 50 psi range, the characteristic time T_2 is at least 0.1 sec and less than

$$0.1 + \frac{0.5}{4(kt)^{1/3}} \text{ (sec)}$$

and possible wave durations T can be calculated for several realistic yields as shown in Table II.

Table II

POSSIBLE WAVE DURATIONS AT 50 PSI

Yield (mt)	0.1	1.0	10
T_1 (sec)	0.46	1.0	2.2
2.5 T_2 (sec)	<1.7	<3.4	<6.7
	>0.25	>0.25	>0.25
T (sec)	<2.2	<4.4	<8.9
	>0.46	>1.0	>2.2

According to Fig. 9 the frequencies most threatened are those between 2/T and 10/T, a range which could conceivably embrace frequencies between 0.2 and 21 Hz. A peak ground speed amounting to 8 ft/sec at the 50 psi range can only be associated with a wave in which the rolling component strongly predominates and its own characteristic spectrum (Fig. S-5) has a narrow maximum between

$$\frac{3}{T} \text{ and } \frac{5}{T} \quad .$$

If T falls between 0.25 and 6.7 sec the endangered frequencies are then between 0.45 and 20 Hz, essentially the same as the range computed above using Fig. 9.

Since peak airblast overpressure and peak ground speed in the ground-transmitted motion both depend on range in approximately the same way, maximum spectral ordinate will decrease in proportion to the peak overpressure; e.g., at 25 psi the highest reasonably "expected" ordinate will be 16 ft/sec and the "proven" ordinate, 4 ft/sec. At 25 psi outrunning is more likely to have occurred or the rolling waveform has a longer duration than at 50 psi. The least possible duration T will be the same as at 50 psi but the largest possible duration will be four times as large.

The occurrence of a particular undulatory shape at two such seemingly widely different environments as Eniwetok and Nevada does suggest a certain degree of universality in the form and it is difficult to see how the extrapolations undertaken here can be regarded as fanciful. It seems conservative to attribute the relative insignificance of the actual transmitted component at the 50 psi range in Nevada to the fact that the explosions were 200 ft/kt$^{1/3}$ or more above the surface and to compute the amplitude of the hypothetical transmitted component by extrapolating the amplitude observed at far ranges back toward ground zero, i.e., to use Eq. (1). As stated before, Eq. (1) stems actually from observations of waves to which the direct airslap made major contributions but see Appendix C for a heuristic justification of this procedure. Thus,

$$V_M = 8.0 \text{ ft/sec} \quad .$$

E. Synthetic Waveform Spectra

Synthetic waveforms for vertical ground motion corresponding to hypothetical nuclear bursts over five actual geologic formations as known from the literature have been devised and speed shock spectra computed from them. The five sites are those examined by Murphy (Ref. 91) for peak motion parameters under the same burst conditions. Synthesis was carried out by simple superposition of the two idealized waveforms discussed earlier in this section. The beginning of the rolling or ground-transmitted component was taken at the time and location of first outrunning as predicted for each particular formation, as illustrated in the accompanying sketch.

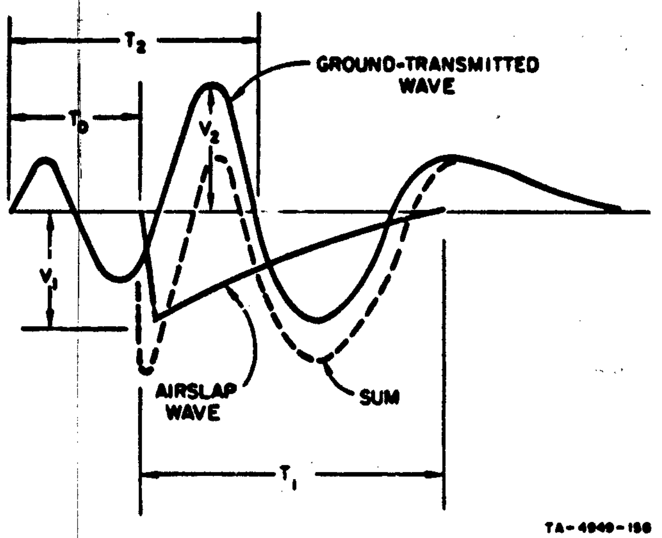


Table IIa shows the parameters characterizing the synthetic waveforms at each of five geologic sites. Time T_1 is duration of airslap at the overpressure range given, i.e., 50, 25, or 10 psi. The waveform of ground motion due to airslap has been assumed to follow the airblast overpressure waveform exactly and the exponential fit supplied by Brode (Ref. 39) was used in the computation of response spectra. Time T_2 in Table IIa is the characteristic time of the rolling component (see Fig. 10). Time T_o is the delay between the beginning of the roll and the beginning of airslap; V_1 the peak downward ground speed in the airslap; V_2 the peak upward speed in the ground-transmitted motion; and A is an estimate of peak acceleration at the onset of the airslap motion. Since the hypothetical bursts at San Jose and Albuquerque occurred at 14,500 ft above ground level, peak airblast pressures above 24 or 25 psi on the ground did not occur for those sites.

To a good approximation the spectra shown in Figs. 12a through 12p are a superposition of the spectra for the two idealized waveforms themselves, as illustrated in Figs. S-3 and S-5. However, peak spectral ordinates due to the rolling motion are generally depressed, apparently due to destructive interference by the downward airslap.

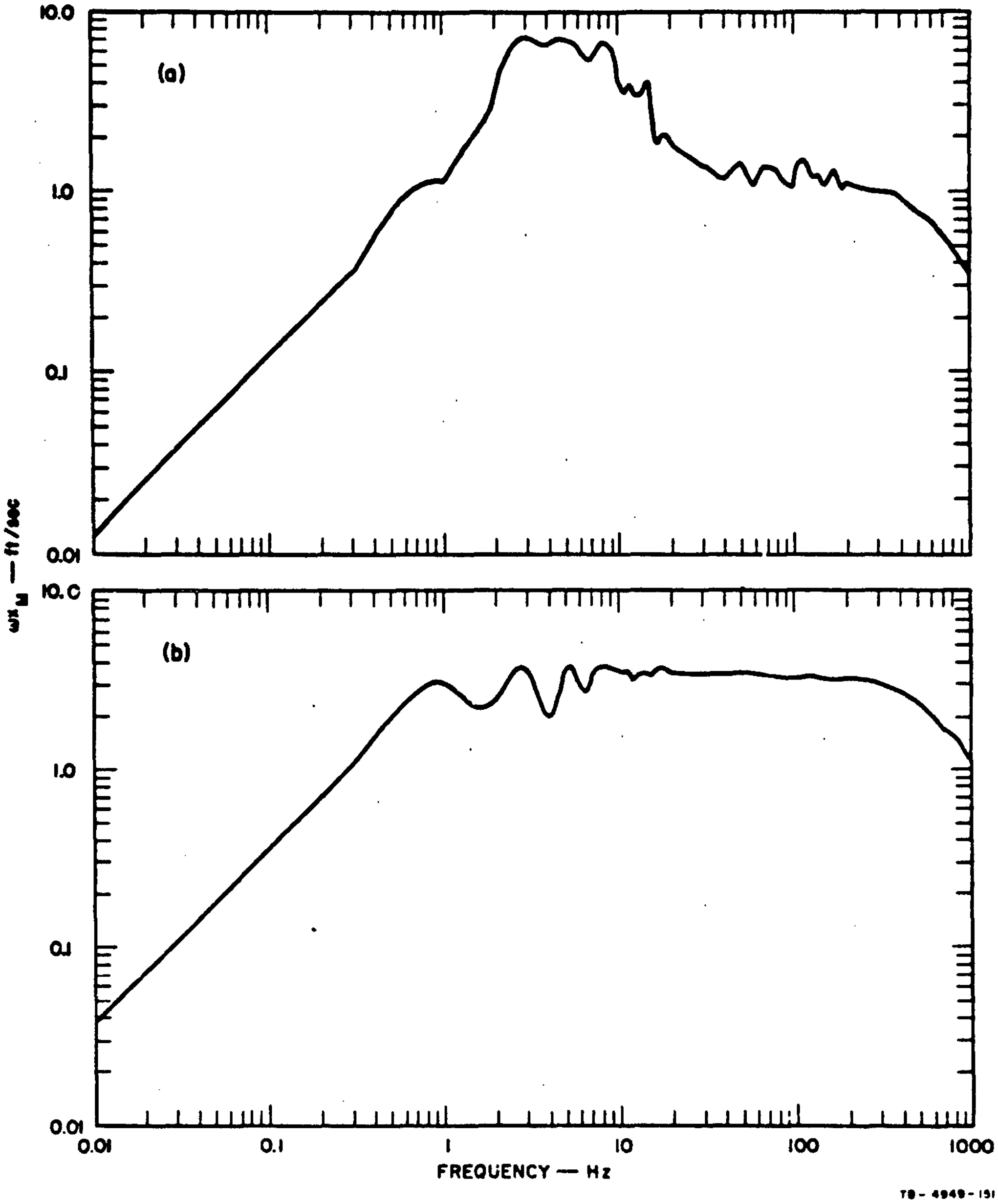


FIG. 12 (a) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, NEW ORLEANS, 50 psi
(b) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, PROVIDENCE, 50 psi

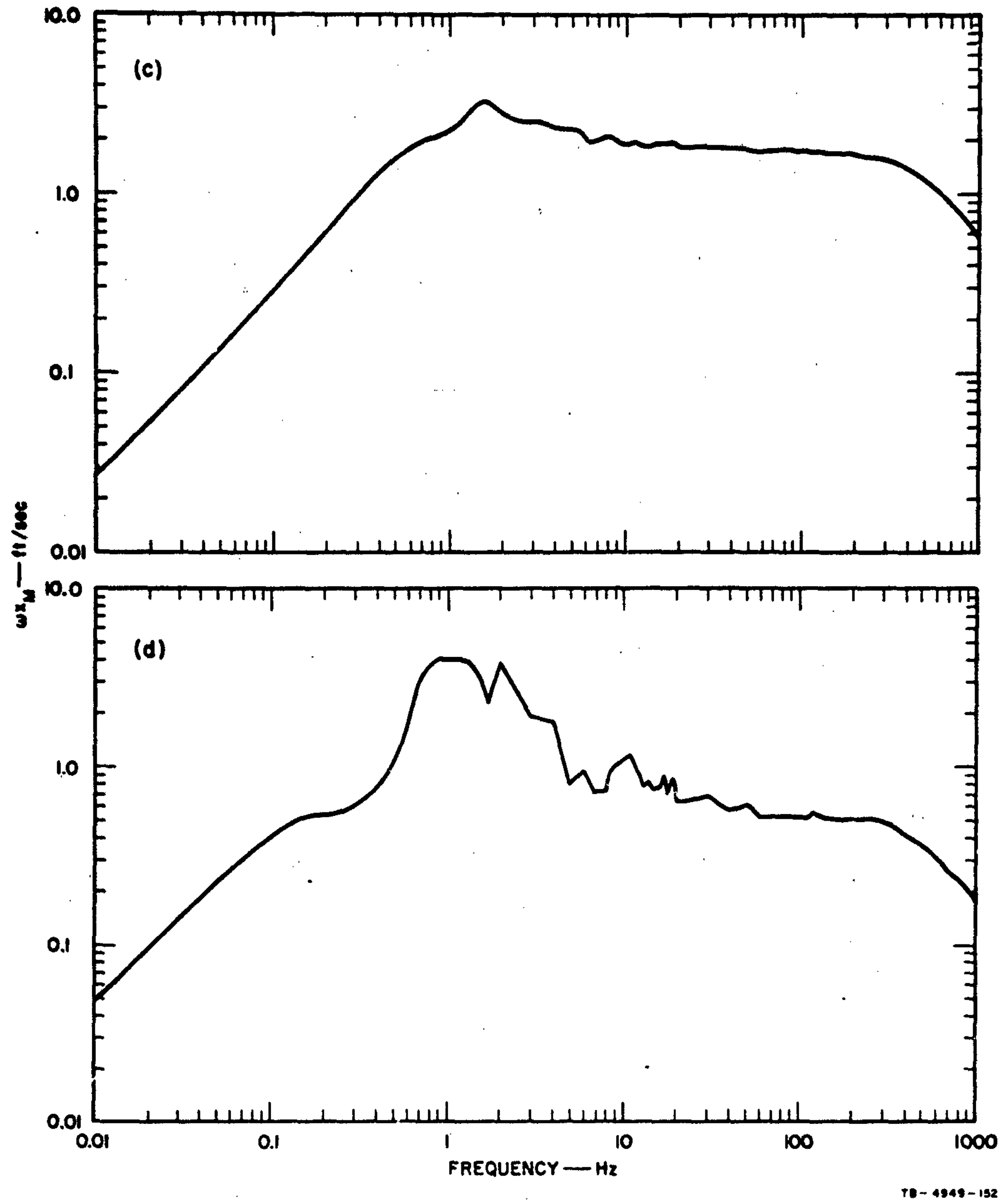


FIG. 12 (c) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, DETROIT, 50 psi
(d) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, NEW ORLEANS, 25 psi

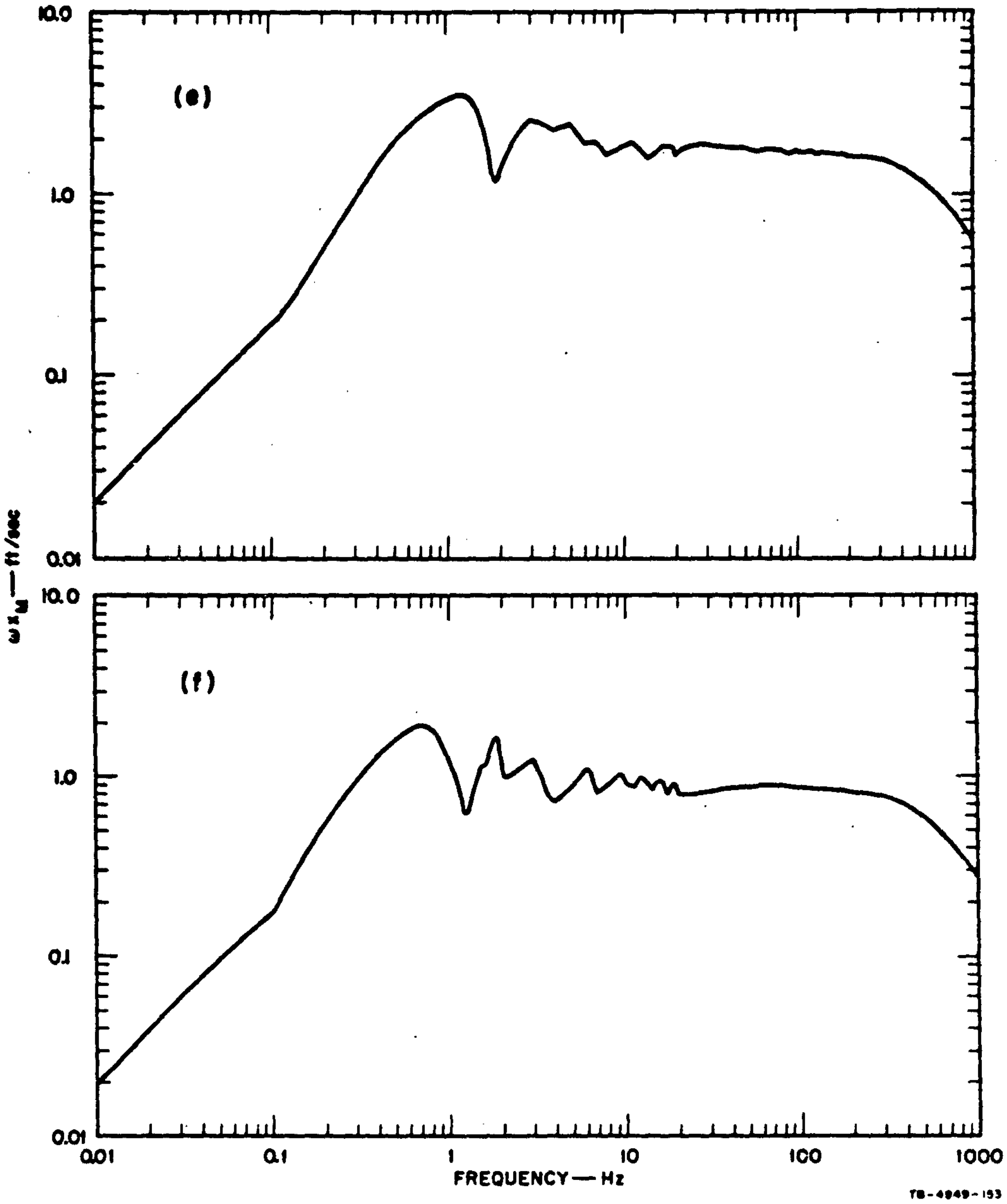


FIG. 12 (e) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, PROVIDENCE, 25 psi
(f) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, DETROIT, 25 psi

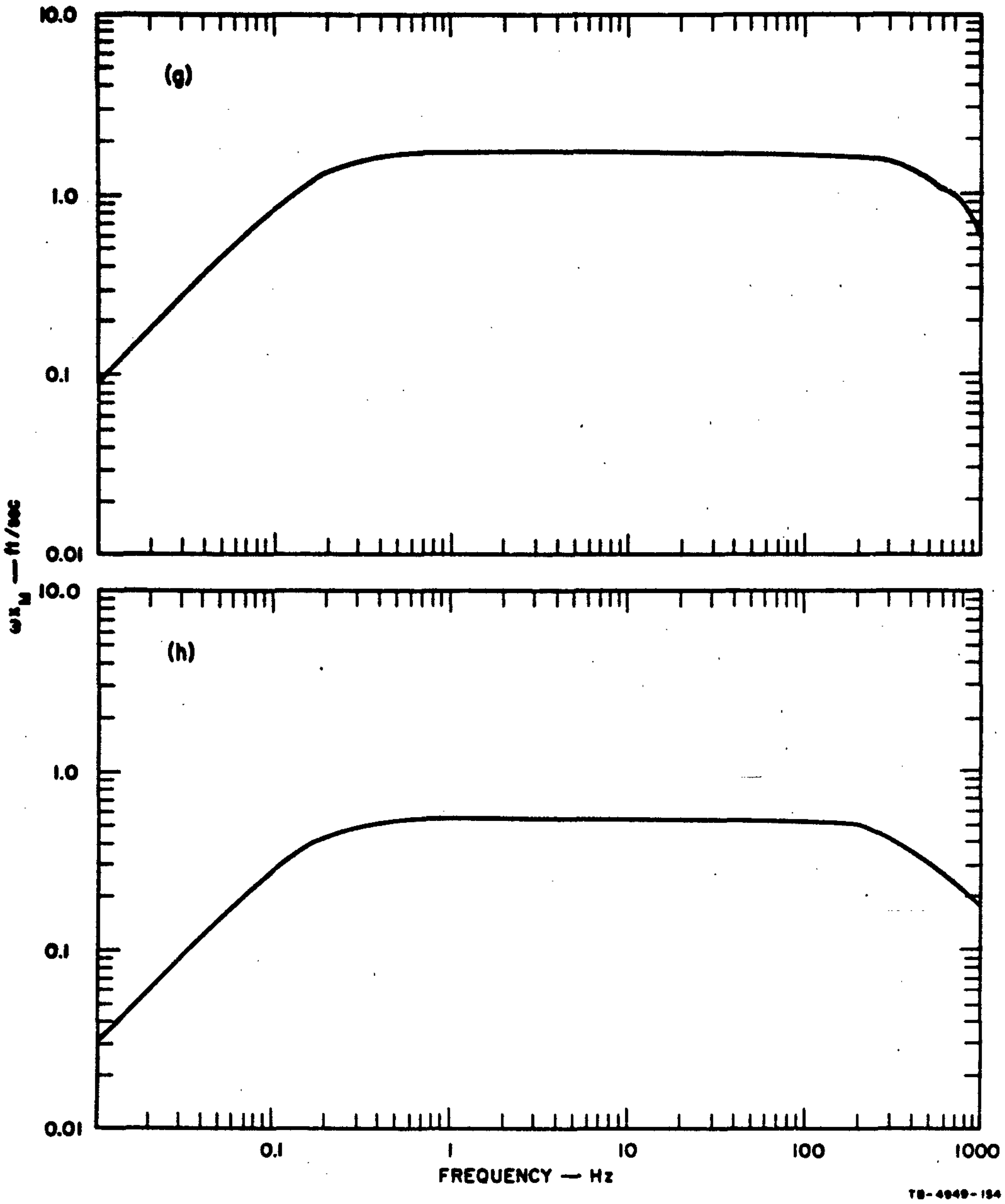


FIG. 12 (g) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, ALBUQUERQUE, 25 psi
(h) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, SAN JOSE, 25 psi

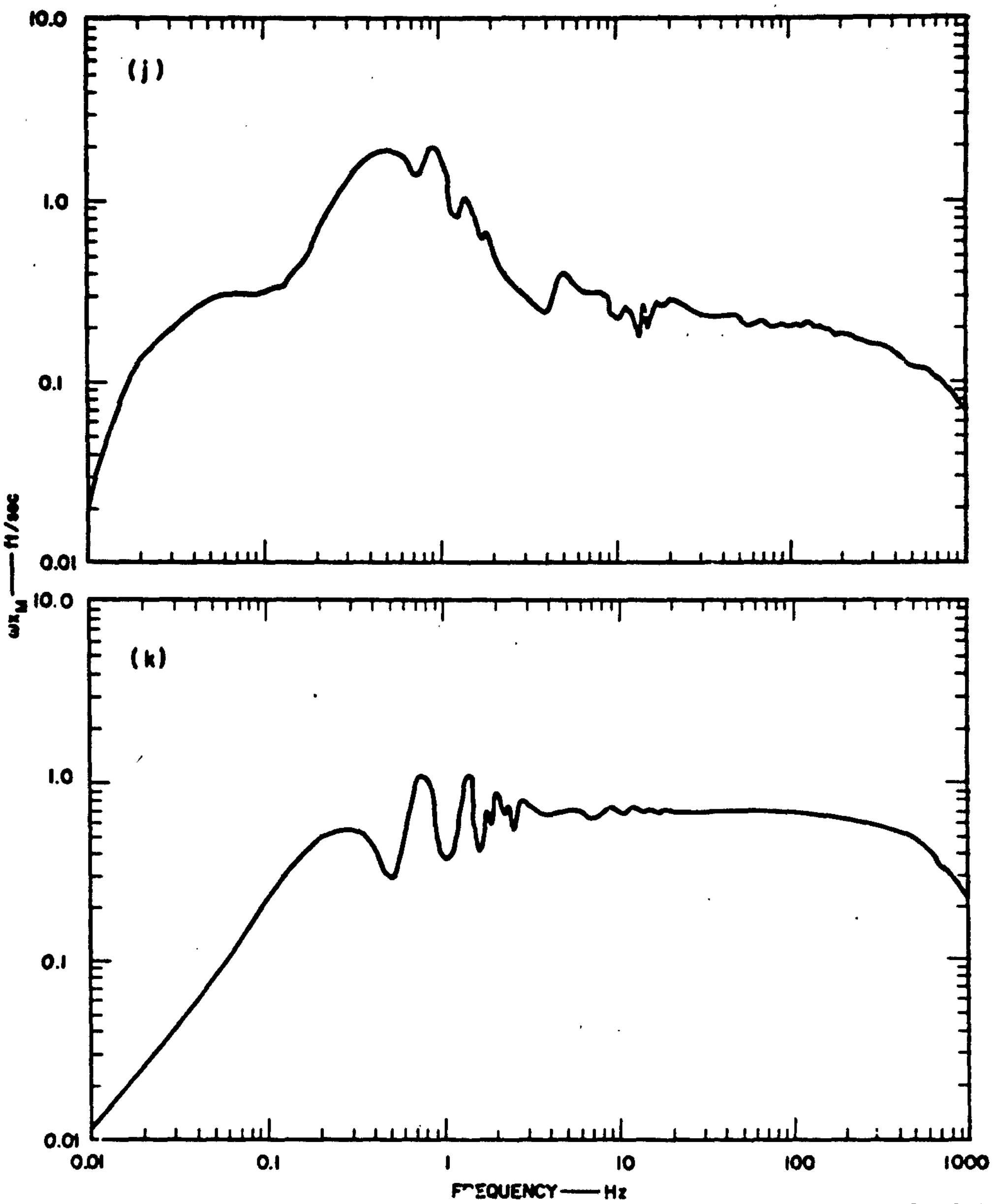


FIG. 12 (j) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, NEW ORLEANS, 10 psi
(k) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, PROVIDENCE, 10 psi

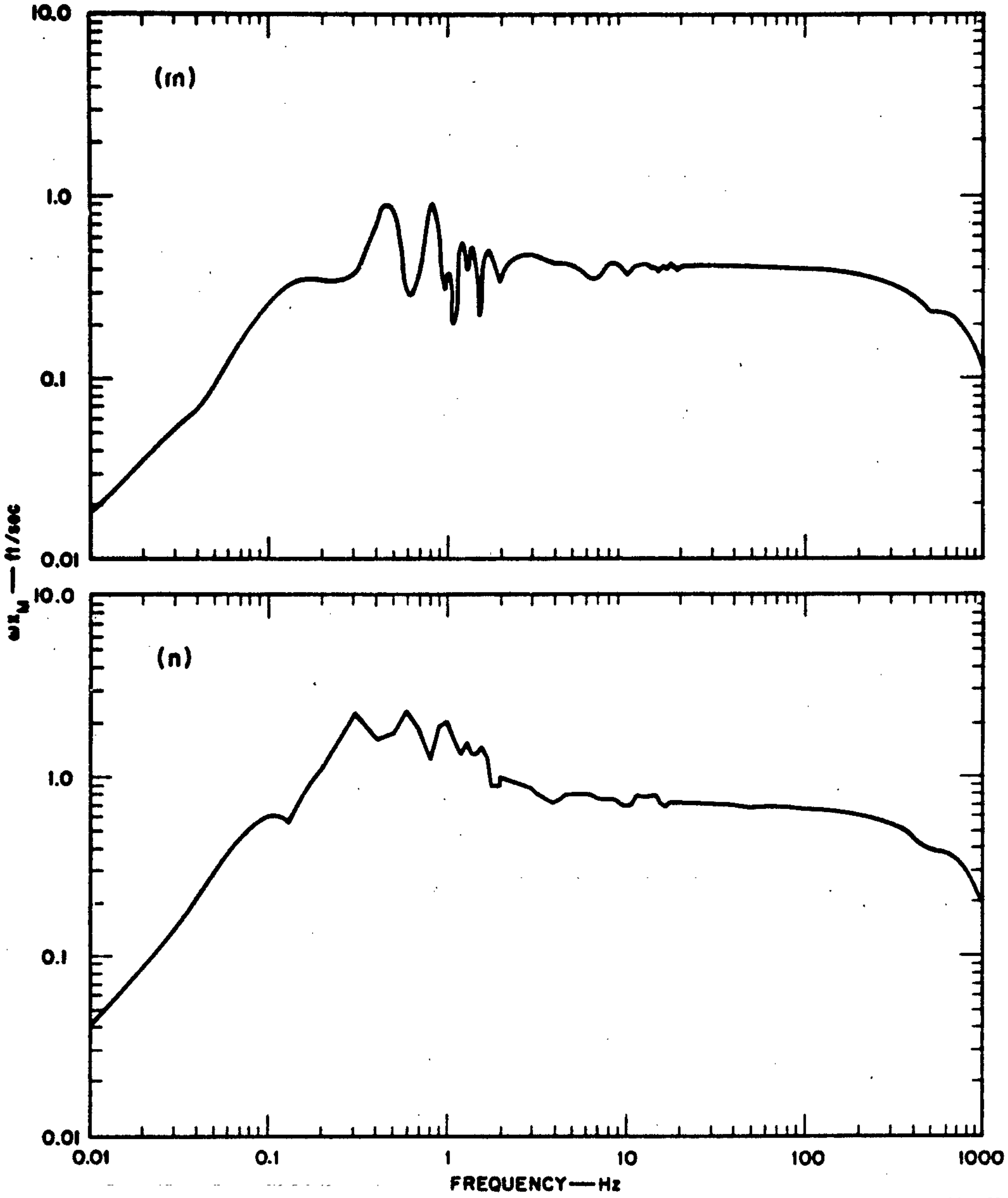


FIG. 12 (m) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, DETROIT, 10 psi
(n) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, ALBUQUERQUE, 10 psi

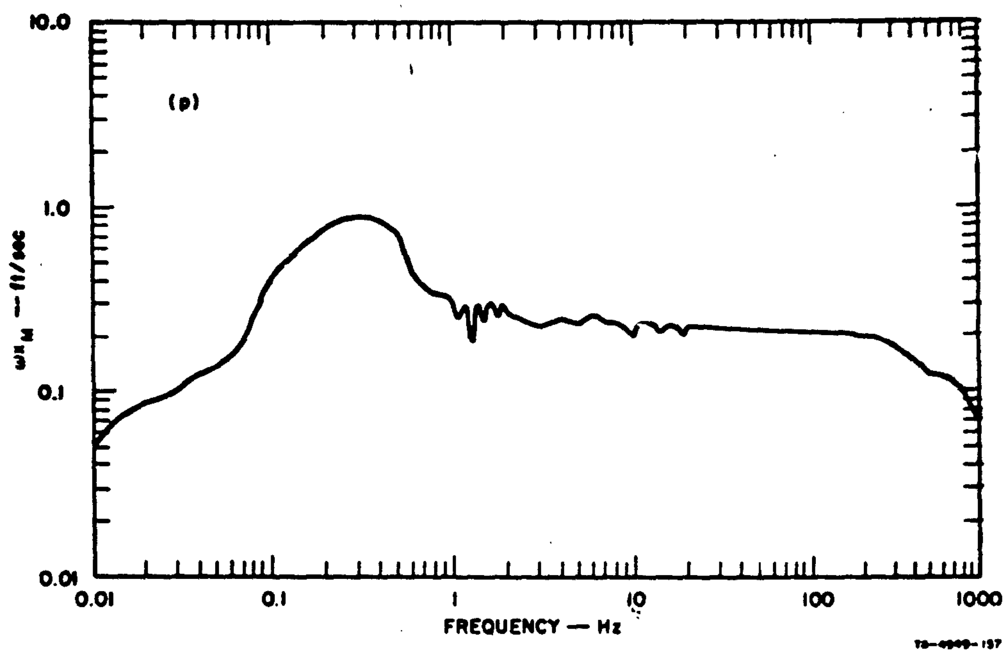


FIG. 12 (p) VERTICAL RESPONSE SPECTRUM CORRESPONDING TO ARTIFICIAL WAVEFORM, SAN JOSE, 10 psi

Table IIa

SYNTHETIC WAVEFORM PARAMETERS
(Vertical Motion)

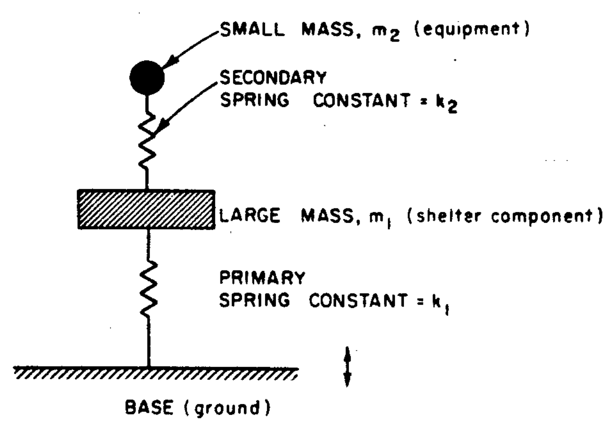
	New Orleans (10 mt, HOB = 0 ft)			Providence (1 mt, HOB = 0 ft)			Detroit (5 mt, HOB = 0 ft)		
	50 psi	25 psi	10 psi	50 psi	25 psi	10 psi	50 psi	25 psi	10 psi
T_1 (sec)	2.11	2.8	4.1	0.98	1.3	1.9	1.7	2.22	3.2
T_2 (sec)	0.35	1.27	2.95	0.66	1.1	2.0	1.02	1.8	3.6
T_0 (sec)	0.00	0.25	1.95	0.53	1.25	3.3	1.0	2.1	5.8
V_2 (ft/sec)	3.8	1.9	1.0	1.14	1.0	0.24	0.75	0.61	0.29
V_1 (ft/sec)	1.0	0.5	0.2	3.3	1.7	0.665	1.7	0.85	0.41
A (ft/sec^2)	1200.	583.	240.	3870.	1920.	767.	1980.	975.	480.

	Albuquerque (5 mt, HOB = 14,500 ft)		San Jose (5 mt, HOB = 14,500 ft)	
	10 psi	24 psi	10 psi	25 psi
T_1 (sec)	4.7	2.7	4.65	3.1
T_2 (sec)	3.2	0.0	6.5	0.0
T_0 (sec)	0.0	0.0	5.9	0.0
V_2 (ft/sec)	1.1	0.0	0.38	0.0
V_1 (ft/sec)	0.67	1.7	0.21	0.52
A (ft/sec^2)	800.	2020.	240.	600.

F. Influence of the Shelter Structure

The people and equipment of concern here will be inside a structure which itself is an object exposed to earth motion. The free-field data from measurements at weapons test are not enough by themselves to describe the pertinent hazard but, when a resonant frequency can be found for the element, they provide only peak values of displacement, speed and acceleration of an element of the housing to which the sensitive equipment or human is attached. To derive from these three values a second response spectrum envelope giving the likely strain on the attached object, Newmark (Ref. 45) suggests extending the empirical observation noted above, that is, constructing the response spectrum envelope of the object attached to the shelter from the peak values of the acceleration, speed, and displacement of the structural element by the trapezoidal rule in the form outlined above for one-sided speed pulses. (The method however is applied to all waveforms.)

In other words, Newmark sees two simple oscillators in tandem; the mass of the first being much larger than the mass of the second and serving as the base of the second as well. The scheme is sketched below. From the peak parameters A_M, V_M, and D_M of the free field ground motion, a trapezoidal envelope of the spectrum for any primary oscillator is found. The frequency f $(= 1/2\pi\sqrt{m_1/k_1})$ of the oscillator (structural element) consisting of m_1 and its spring is calculated and peak values A_M', V_M', and D_M' read from the response spectrum envelope at the abscissa f.



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(For an element partially embedded in soil this calculation may not be straightforward.) Since V_M and D_M refer only to the motion of m_1 relative to the ground and A_M' is absolute acceleration,* the peak values of the motion parameters of m_1 as a platform will conservatively be taken as acceleration, A_M', and the sums, speed $V_M + V_M'$, and displacement $D_M + D_M'$. Thus the envelope of the response spectrum defining the hazard to the secondary system is defined by the trapezoidal rule as:

$$\text{max acceleration} = \omega^2 x_M = 2A_M'$$

$$\text{max speed} = \omega x_M = 1.5\ (V_M + V_M')$$

$$\text{max displacement} = x_M = D_M + D_M' \quad .$$

The actual maxima of the parameters of the secondary system, i.e., A_M'', V_M'', and D_M'' will be read from the spectrum so defined at the secondary resonant frequency $f' = 1/2\pi \sqrt{m_2/k_2}$.

When the frequencies of primary and secondary oscillators are near each other, the response of the secondary can exceed that given by the envelope constructed as above. Without damping, this response can become infinite. In any real case, of course, damping is present and Newmark provides certain approximate procedures for handling those cases when the two harmonic oscillator frequencies are close (see page 45 of Ref. 45). The result of these procedures is to raise a bump on the trapezoidal envelope for the secondary system in the neighborhood of $f \approx f'$. Newmark

* If y and z are absolute displacements of ground and mass, respectively, and x is change in length L of the undamped spring, then

$$m\ddot{z} = -k(z - y - L) \qquad \text{or}$$

$$\ddot{z} = -\omega^2 x$$

Hence $\qquad \max |\ddot{z}| = \max |\omega^2 x|$

defines the neighborhood of $f \approx f'$ as $1/2f < f' < 2f$ and in that region multiplies the ordinate of the envelope by the factor:

$$\alpha = \frac{1}{1 - \left(\frac{f'}{f}\right)^2} ,$$

whenever $\alpha \leq 1/2\beta$ and by the factor $1/2\beta$ when $\alpha > 1/2\beta$. The quantity β is the proportion of critical damping in the secondary system.*

Agbabian-Jacobsen Associates (Ref. 33) report natural frequencies of walls and floors of certain buried structures. For all below ground elements, except one, these frequencies fall in the range 20 to 55 Hz. The exception is important, a floor under a steel arch buried 25 ft below the ground which vibrates at 5 Hz, a frequency of great potential hazard to the human body. The floor and wall of a 1.3% reinforced concrete rectangular box 20 ft × 20 ft in cross-section vibrate at 24 and 25 Hz, respectively; slab thickness is 2 ft 6 in. and the box is buried so that its rooftop is flush with the ground surface. The authors state that damping amounting to 5% of critical was assumed for all structural elements backed by soil; this would result in only a slight shift in natural frequency but is seemingly the only soil-structure interaction considered. The authors further indicate that 5% critical damping was assumed for all secondary systems; however, the response spectra envelopes included in their report show peak amplification of approximately 2.5 instead of

$$\frac{1}{2\beta} = \frac{1}{2 \times \frac{1}{20}} = 10$$

as expected from the Newmark formula.

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* These ideas appear to come from the amplitude ratio of an undamped two degree-of-freedom tandem oscillator under steady sinusoidal excitation, derived on p. 332 of Ref. 5, and from the amplification ratio of a single degree-of-freedom damped oscillator also under steady sinusoidal excitation, treated on p. 218 of Ref. 5. The use of these formulas based on steady sinusoidal excitation is conservative for transient inputs.

It should be noted that the maxima ωx_M and x_M defined above are of course always greater than the corresponding values for any primary oscillator in the free field, viz., V_M and D_M, and the result of the interposition of structure between equipment and soil is always to raise the hazard defined by two of the three sides of the trapezoid corresponding to any secondary oscillator response.

The peak absolute acceleration A_M'' can on the other hand in theory be less than free field or ground maximum acceleration. In this case the housing structure filters out high accelerations. However, any ground wave carrying extremely high acceleration will probably be transmitted through the structure to the occupant more as a shock wave than as motion of a whole wall or floor and the harmonic model will not be appropriate in discussing peak accelerations.

Actual peak speed and displacement (relative to the primary mass) in a secondary system depends on wave shape. The question of whether or not the arithmetic addition used above to bound these values is overly conservative for nuclear induced motion is not clearly resolved from weapons test data. Comparisons of free field and indoor displacement spectra have been observed, but other factors influence the results. For the best comparisons the two measurements should be made at the same depth and the inside measurement should be made at the point on the structural element where movement is likely to be the greatest. And the structural element should be large enough to have natural frequencies of interest. In order to observe displacement spectra, Halsey and others (Ref. 46) bolted reed gages to the concrete floor of earth-covered quonset huts at Eniwetok as well as buried the same gages in the free field just below the surface. There were two explosions, one approximately 20 kt, the other between 1 and 2 mt.

Within one foot of the surface the ground motion wave shapes at both shots were predominantly one-sided (Ref. 41). The size of the concrete slab appears to have been in the range 10 X 10 ft to 20 X 20 ft; Halsey does not report the thickness. The slab seems to have been on the original ground surface. The reed gages were not placed in the center but near the middle of one edge. On both shots, inside and outside dis-

placements can be compared at ranges corresponding to values of peak overpressure between 85 and 90 psi. The results in both vertical and radial directions show neither magnification nor attenuation of peak displacement between 2 and 10 Hz, but an undoubted attenuation inside the huts at frequencies above 10 Hz. The observed spectra both inside and outside are consistent with trapezoidal envelopes in the $(\omega x_M, \omega)$ plane; and the high frequency asymptote of the observed indoor vertical spectrum can be predicted from the free field spectrum on the foregoing rules provided the predominant natural frequency of the vertical slab motion is taken in the range 4 to 6 Hz. The only apparent effect of weapon yield is to lessen the discrepancy between inside and outside displacements when the yield is lowered.

Somewhat contrary indications come from the 116-psi overpressure range at Operation Plumbbob (Ref. 46) Nevada Test Site. Here spectra were taken with gages near the surface in the free field as well as bolted to the floor slab of a wholly buried reinforced concrete shelter. The slab was 2 ft thick, over 13 ft below the ground surface, and about 20 X 10 ft in area. Horizontal spectra were essentially the same indoors and out. Compared to free field data the vertical spectrum showed uniform attenuation throughout the frequency range 2 to 200 Hz; that is, in contrast to the Eniwetok behavior high frequency gage responses are not reduced compared to those of low frequency gages. The overall lowered response on the floor can most easily be attributed to the depth of burial. The seeming contradiction between the conclusion from the Plumbbob results and that from Eniwetok may be due to the existence of a higher natural frequency in the heavy reinforced shelter slab in the quonset hut floor, as demonstrated by the following argument. For a structure similar to the reinforced underground shelter used at Plumbbob, Agbabian-Jacobsen compute a frequency of 24 Hz (Ref. 33). The observed vertical free-field velocity spectrum (near the surface at 116 psi) from Plumbbob can be enveloped by asymptotes $\omega x_M \cong 2\pi$ ft/sec and $\omega^2 x_M \cong 50$ g (see Fig. 3.123, Ref. 47). The intersection of such an envelope with the abscissa f = 24 Hz takes place at $\omega^2 x_M$ equal to about 24 g. Since, according to the rules given above, one edge of the envelope of the

secondary system is located by twice this value, viz., 48 g, the high frequency asymptote of the indoor spectrum should fall near the same asymptote for the free-field spectrum and differential indoors attenuation of high frequency response over low frequency response does not occur. (Reduction of the value $\omega^2 x_M \cong 50$ g to account for depth of burial will not change the argument essentially.) The reed gages at Plumbbob, too, were not placed near the middle of the slab but toward one wall of the shelter.

We conclude that while the likelihood of shrinking a free-field response spectrum envelope in the high frequency region by the interposition of a structure has been confirmed experimentally, there is no evidence of a corresponding magnification in the low frequencies. However, because tests have been limited, the chance of magnification must be considered. In particular, we have no experimental evidence from purely rolling wave motions.

If spectra of secondary systems are derived by addition of base motion then clearly the presence of housing may magnify the hazard; and if a spectrum envelope is defined by $4V_M$ (as in Fig. 9) instead of $1.5\ V_M$ or if the effect of equipment damping is more like that assumed in Ref. 46 than that computed in Ref. 34, then the magnification at certain frequencies can be an order of magnitude or more. Undoubtedly structural response is itself a potential source of danger and it is unfortunate its quantitative analysis remains inadequate.

G. Likely Sources of Human Injury

When the ground shock from a nuclear explosion arrives at the shelter, a person may be standing, sitting, or lying in contact with a wall or floor. The thrust of the heavy structure against him may violently displace one part or parts of his body with respect to others to an injurious extent. The person may be thrown against a heavy or sharp object with enough speed to cause injury. He may simply be surprised by the shock, lose his balance, and fall against a heavy or sharp object.

There was some concern during World War II that dust or missiles spalled off concrete shelter walls by the impinging shock wave constituted a hazard to people in the shelter, but research both during World War II and after indicates that this danger cannot be great (Refs. 3, 47, 48). It is assumed in present study that common sense precautions have been taken to keep heavy bookcases, pictures, mirrors, or other lightly fastened or breakable fixtures off the shelter walls.

VI DATA ON HUMAN TOLERANCE TO IMPACT

Under the stimulus of high speed flight, the space program, and the mounting traffic toll, much investigation has been carried out in the past twenty years on the effect of vibration, acceleration, and impact on man and animals. For present purposes, results of this work can be considered under two headings: (1) effects of relative displacement of internal and external organs, and (2) effects of shock waves. The experimental studies which can contribute to an understanding of the first category are horizontal deceleration of tightly harnessed human volunteers and animals on sleds and drop tests of free standing or sitting volunteers. Also of value are studies of falls which result in nearly simultaneous impact of all parts of the body with a yielding material. Obviously it has been impossible deliberately to explore the human injury threshold in these experiments and the definition of such a threshold for various experimental animals is of limited applicability to humans. Much of the work on the effects of shock waves has again been done with human volunteers and the limits found have been those of voluntary tolerance. However, there are some reports of pertinent experiments with fresh human cadavers and studies of impact accidents involving humans. The parts of the body most sensitive to this kind of damage are the head and the heels.

A. Supported or Whole Body Impact

For the inner organs or tissues of the body the skeletal framework is a base in the sense of the simple harmonic oscillator model described above. Much experimental work has been reported in which the motion of this base was known and impact tolerance limits discovered.

1. Transverse Motion

a. High-Speed Sled Impacts

Stapp performed two series of tests in which tightly harnessed, seated male volunteers on sleds were propelled along the ground and brought to a sudden halt (Refs. 12 and 13). The motions of both the seat and the skeleton were monitored; and both deceleration pulses were generally trapezoidal in shape. In the first series of tests speed change, v, varied from 210 to 70 ft/sec; deceleration pulse duration fell between 130 and 350 msec. The subjects were seated upright facing toward or away from the point of impact. The most severe injury produced was a mild, temporary physiologic shock. There was some dependence on pulse shape; that is, the faster-rising acceleration pulses were associated with the more severe symptoms. In Fig. 13, which is a graph of

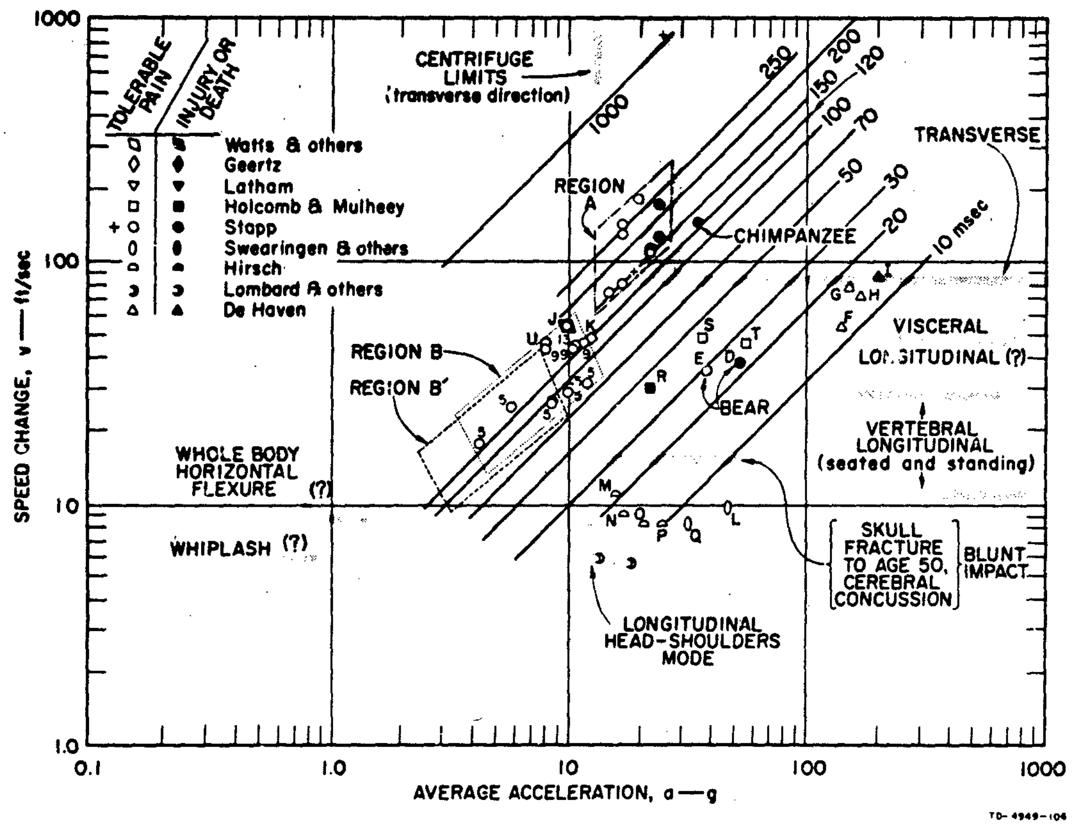


FIG. 13 HUMAN TOLERANCE LIMITS TO IMPACT

tolerance limits in coordinates of v (ordinate) and average acceleration a (abscissa), the range explored by Stapp's first series of experiments has been marked "Region A." Data points corresponding to experiments which produced (reversible) injury are shown solid; others are open. His second series of tests were designed to look into the region 20 ft/sec $\le v \le 45$ ft/sec, $60 \le T \le 130$ msec marked "B" in Fig. 13, but there was considerably more variation in the position of the subject at impact than in the earlier series. Both forward- and backward-facing subjects were reclining in a chair tipped from the upright position (pitch) and in some cases the impact was not directly headward or backward (yaw). No tests were done however with subjects facing in a direction 90^0 to their line of motion, nor was there any rotation about the line of impact (roll). Amount of pitch (from upright seated position) varied from $+45^0$ to -45^0 while both forward- and backward-facing subjects may have been yawed away from the line of impact as much as $\pm 40^0$. The values of a and v shown in Fig. 13 for these pitched and yawed subjects were measured in a direction parallel to the line of impact.

Obviously the differences in bodily positions at impact meant that the kinds of bodily strains produced in experiments falling in region A of Fig. 13 were different from those studies in region B, but since the direction of the earth motion engulfing a shelter will not be known, both kinds are important for present purposes. It is not suggested of course that a person not harnessed or tightly attached to the floor or wall of a moving shelter would move with the floor or wall; the actual motion of his skeletal frame would have to be deduced separately, but the information summarized by the areas A and B in Fig. 13 would then be of help in determining the effect of that motion on the whole organism.

Stapp (Ref. 14) summarizes his first series of experiments by giving three deceleration waveforms or histories which he says are limiting for harnessed, upright-seated men braked from linear horizontal motion. One is triangular and has an average of about 19 g; the second is trapezoidal with an average 27 g; and the third is more complicated in shape but has an average between 25 and 30 g. Although Stapp attributes the peculiar danger of the first to rate of rise (onset) of deceleration

(1500 g/sec), of the second to peak deceleration (48 g), and of the third to duration (1.1 sec), his summary of the hazard in transverse deceleration is consistent with a vertical asymptote passing the neighborhood of the right edge of region A in Fig. 13, viz., within the bound $20 \leq A_o \leq 30$ g. (Points corresponding to each of Stapp's three limiting wave shapes are shown as crosses in Fig. 13). Stapp reports no appreciable difference between the forward- and backward-facing positions in this series of experiments.

Stapp's early work did not establish a limit V_o for pulses of very short duration and very high acceleration. This asymptote will be located by reference to data from free falls.

b. Free Falls

De Haven (Ref. 21) has quantitatively studied certain free falls accidentally or suicidally undergone by humans, most of whom survived apparently uninjured. From the height of fall de Haven estimated speed and from depth of dents or depressions in the struck surface he found average deceleration. Because survival was accompanied except in one case by a nearly simultaneous contact all over the body with a yielding material, these data furnish examples of deceleration of the supported body. Again the range of bodily positions at impact was limited; de Haven's cases were either prone, supine, or nearly so at moment of impact. There was apparently no clear example of purely side impact. In Fig. 13 point F corresponds to uninjured survival after a supine fall into packed earth; the subject of datum point G fell prone into the hood and fender of a car. (His only injury--a skull fracture--most likely arose after bouncing to pavement.) Case H broke her rib and wrist when she struck soft earth in a supine position and Point I marks a death from supine impact onto a wooden roof. Death was attributed to severance of brachial arteries, internal bleeding, and shock. Thus for prone or supine supported whole body impact de Haven's data would indicate a V_o near 80 ft/sec. However, this is not likely to be a level of strain which would easily be borne willingly.

If the long duration tolerance limit is taken at $A_o = 25$ g and the short lasting boundary as $V_o = 80$ ft/sec, then $T_o = V_o/A_o = 100$ msec and the natural frequency of the equivalent oscillator is in the neighborhood of

$$\frac{1}{4} T \le T_o \le \frac{3}{4} T \quad \text{or} \quad 2.5 \le f \le 7.5 \text{ sec}^{-1}$$

The tolerance limits shown in Fig. 13 labelled "transverse" are essentially the same as those suggested by Kornhauser and Lawton (Ref. 34).

More recent data on transverse impact at high acceleration comes from Holcomb (Ref. 49) who observed acceleration of a man dropped downward onto a hard surface on his back. Pulse shapes were triangular, 40 msec long and up to 73 g peak. There were no injuries. Datum point S, Fig, 13, taken from this series is clearly within the zone of tolerance. Holcomb also tested men in sideward impact. These subjects were again dropped but also had horizontal speed. Accelerograms show transverse components roughly triangular in shape, 15 to 30 msec long, 100 to 115 g peak. A representative datum appears in Fig. 13 as point T. There were no injuries.

c. An Auto Collision Case

An auto crash case reported by the Automobile Crash Injury group of Cornell University supports the validity of the limit $V_o = 80$ ft/sec for transverse (fore and aft) impact. A driver was stopped from a speed of about 55 mph (80.5 ft/sec) by the (non-telescoping) steering wheel of his car when his car hit a concrete abutment (Ref. 50). Since the lower half circle of the steering wheel was bent almost vertical, and since the man's body very likely struck an essentially motionless wheel, stopping distance was in the range of from 3 to 6 inches and average deceleration of the chest must have been over 100 g. As would be expected from local or incompletely supported nature of the impact, there was rib breakage, but no discernible visceral harm. (Such bone fracture will be discussed as shock wave damage.)

d. Sidewise Impacts

Aside from the few human experiments by Holcomb noted above, the only data relating to side-on impacts appear to be that of Robinson, Hamlin, Wolff, and Coermann (Ref. 51), who dropped rhesus monkeys in half body molds while monitoring accelerations of platform as well as various body parts. Their subjects survived all tests with at most only minor, temporary injury. Data points in the coordinates of Fig. 13 would fall roughly along a line defined by the two end points

$$(T = 70 \text{ msec}, V = 5 \text{ ft/sec})$$

and

$$(T = 30 \text{ msec}, V = 30 \text{ ft/sec})$$

Stapp (Ref. 14) mentions side-on impact of chimpanzees and hogs, but cites no data for this position.

Dieckmann in his work on horizontal oscillation of men (Ref. 25) remarks only that motion of seated and standing subjects in the frontal bodily plane (viz., sidewise motion as opposed to fore and aft movement) does not typically give results different from those associated with motion in the sagittal or fore and aft plane.

It does not appear that sidewise transverse impact is greatly more or less hazardous than prone or supine.

e. Transverse Vibration of Human Body

So far as is known only one study of the response of prone or supine men vibrated vertically has been attempted. Dieckmann (Ref. 10) reports merely that investigation of lying subjects could be carried out only up to a frequency of 5 Hz because the subjects always found the oscillation more "unpleasant" in that position than standing or sitting. He gives no other results from observations on these subjects; but in another later investigation (Ref. 51) he placed unsupported seated and standing men on a platform oscillated horizontally at frequencies in the range 0.4 to 50 Hz. Although the accelerograms obtained at different parts of the body of these subjects would certainly show the influence

of flexure in the spine, legs, and neck, they may also be influenced by bodily motions which are important in determining man's tolerance to flat impact as discussed by de Haven. For example, in the frequency range from 1 to 2 Hz both the hip and knee bones of seated men shaken fore and aft by Dieckmann amplified the table acceleration and moved with a phase displaced $\pi/2$ from that of the table. At higher frequencies the phase shifted to π and at lower frequencies these parts were in phase with the table. Amplitude of motion also fell sharply on both sides of the resonance, as is typical of behavior of an oscillatory system near onance. Dieckmann also finds a broad, weak resonance in the range from 3 o 5 Hz in accelerograms placed in close contact with the hip bones of men standing on the horizontally shaking platform. Although he is silent on the meaning of the resonance in seated men between 1 and 2 Hz, Dieckmann interprets the behavior in the hips of standing men at 3 Hz as stemming from a second flexural or bending mode in the whole body. Treating the subject as a uniform bar about 6 ft long shaken transversely on one end, he calculates a second eigenfrequency of 2.8 Hz. Since the equivalent "length" of a seated man is less than that of a standing man, the resonance between 1 and 2 cps cannot be a simple bar oscillation. The influence of a secondary oscillator attached to a primary is to shift the primary frequency away from the secondary frequency and to introduce a second resonance, which itself is shifted from the frequency of the secondary oscillator in a direction away from the primary. [In Ref. 5 (Chapter 7) a few examples which illustrate this behavior are computed.]

It can be argued then that hip motion is influenced by the response of heavy viscera attached to the skeleton and that it is the strain on linkages between inner organs and skeleton that is associated with the injuries in flat impact mentioned above. Internal injury in cats due to falls has been established by Rushmer, Green and Kingsley (Ref. 52); Aldman has actually photographed with x-rays internal motion response in hogs (Ref. 53).

2. Longitudinal Motion

a. High-speed Sled Impacts

Because he tilted some of his sled riders 45° to the horizontal, Stapp provides data on tolerance to deceleration parallel to the spine. Some of the data has been plotted in Fig. 13 in the area marked region B. The symbol "5" means that the point refers to a forward-facing impact; "9" and "13" indicate a backward-facing deceleration; for points labelled 5 and 9 the subject was also upward facing but for 13 he faced downward. The experiments represented by points on the right hand edge of the region produced considerable pain to the subjects but no irreversible injury. The backward-facing impacts in region B were markedly less painful, probably because of the better body support given by the chair back than by the chest straps.

In region B the indicated values of v are sled impact speeds and the values of a are one-half the stated peak sled deceleration along its track. Thus to estimate actual longitudinal parameters v and a on the subjects the whole region may be moved downward and to the left in Fig. 13 to account for multiplication of all coordinates by $1/\sqrt{2}$. Region B′ represents region B so transformed. This puts the vertical asymptote for moderate pain below A = 10 g. Some early students of pilot ejection from high speed aircraft reported accelerograms for upward movement of a seated man indicating a higher tolerance level. Two data stemming from this work marked J and K in Fig. 13 tend to move the vertical asymptote near to 13 g. Since it was not uncommon for pilots to be injured by the ejection process points J and K should be taken as limits.

b. Free Falls

There is doubt about the location of the V_o asymptote for impact of seated men. Since the various bodily parts, viscera, spine, rib cage, etc., are not rigidly articulated, two or more bodily sub-systems are very likely involved. The most pertinent direct experimental data seem to be those of Swearingen, McFadden, Garner, and Blethrow (Ref. 20), who dropped unrestrained seated men onto a platform and recorded accelerograms on the platform and on the subjects' shoulders. Drop height was increased until

the subject felt severe pain in his chest, spine, head, and stomach and was diagnosed as suffering from severe, general shock. Although the platform waveshape is not one-sided (but is a well-damped oscillation near 100 Hz reaching peaks between 65 and 95 g) the coordinates (a,v) from the first cycle are plotted as point L in Fig. 13, viz., a = 47 g and v = 9.6 ft/sec. The nature of the pain suggests both bodily systems were strained almost to their limits in the impact. The accelerogram (peak 10 g) at the shoulder suggests a period of oscillation about 0.14 sec in length or a natural frequency near 7 Hz. Swearingen also studied impacts of men dropped with legs flexed for which posture he found a greatly increased tolerance level, but in view of our underlying assumption of surprise these data have not been included here.

c. Ship-shock Simulator

Using a ship-shock simulator Hirsch (Ref. 19) jolted unrestrained seated men upward with increasing intensity until the subjects became uncomfortable and were reluctant to accept higher stress. There were no observable injuries. His data measured on the platform appears in Fig. 13 as points M, N, O, and P. Hirsch also observed peak displacement of the rib cage in relation to the platform slightly less than 40 msec after upward impact of a seated subject. Taking this as 1/4 of the natural period we compute a natural frequency $f \sim 6.2\ sec^{-1}$, in agreement with Swearingen's shoulder measurements. Although the wave shapes of Hirsch and Swearingen undoubtedly differed, both yielded similar values of f_n and asymptote V_o. Hirsch reports data in terms of the upward thrust alone. However, in many cases the subject underwent a second impact when he again met the by then stationary platform. Since platform speeds did not exceed 7 to 9 ft/sec, Hirsch's subjects seem to have been exposed to about the same landing impact as were Swearingen's. Hirsch does not report accelerograms for the upward motion but suggests that the platform moves upward at nearly constant acceleration for about 10 msec then downward at a much lower constant acceleration for about 40 msec.

d. Location of the Asymptotes

Coermann, Ziegenmecker, Wittwer, and von Gierke (Ref. 24), observing abdomen flexure and air movement through the mouth in supine subjects shaken horizontally seem to have found a purely visceral resonance between 3 and 4 Hz.

Choosing A_o = 10 g, natural frequency $f_n = 3.5\ sec^{-1}$, and using the formula $T_o = (1/2)\ (1/f_n)$, the asymptote for longitudinal visceral strain becomes V_o = 45 ft/sec. Since this calculation is speculative, the asymptote has been entered in Fig. 13 with a question mark. Using the wider limits $(1/4)\ (1/f_n) \leq T_o \leq (3/4)\ (1/f_n)$, we compute $25 \leq V_o \leq$ 60 ft/sec. Raising the value of A_o, as seems reasonable, would of course increase the visceral longitudinal speed asymptote.

On the other hand, choosing frequency $f_n = 6.5\ sec^{-1}$ and A_o = 10 g, we find 11 ft/sec $\leq V_o \leq$ 28 ft/sec, where it has been entered in Fig. 13 as the "vertebral longitudinal" asymptote.

Holcomb and Huheey (Ref. 18) irreversibly compressed the T-3 vertebra of a harnessed, seated volunteer who struck the ground with a speed vector inclined about 45° to the horizontal. Since the subject's back was parallel to the ground and head facing upward, the position was an upside-down version of one of Stapp's. Accelerograms of the subject in both longitudinal and transverse direction are roughly triangular, with peaks at 43.5 g, durations 44-55 msec. The longitudinal parameters (a,v) are plotted in Fig. 13 at point R. Holcomb and Huheey state that other tests produced higher transverse or longitudinal accelerations without injury; the authors seem to associate this injury with the direction of the impact upon the spinal column, as well as the impact magnitude. However from accelerograms published elsewhere by Holcomb (Ref. 49), it does not seem that the longitudinal loading ever greatly exceeded that represented by point R, which is clearly in the hazardous zone near the edge of toleration for vertebral longitudinal strain.

One of Stapp's volunteers suffered excruciating lower thoracic back pain when he underwent lower longitudinal shock levels than Swearingen's and Hirsch's men (see run 1559, Ref. 13). However, the input motion was

apparently compound; that is, an unintentionally loose harness allowed a second impact to occur before relaxation of tension from the first had taken place. Since the second pulse started about 70 msec after the beginning and about 25 msec after the end of the first, failure to relax is consistent with a natural period of 1/6.5 sec = 155 msec.

Two of Stapp's experiments with seated black bears (Ref. 13) seem to contradict the tolerance levels assigned above. Components of speed change and average deceleration along the longitudinal body axis have been marked on Fig. 13 as points D and E. The experiment corresponding to D produced a vertebral fracture, lacerations, and bleeding in spleen and liver and some swelling, inflammation, and bleeding in lungs. The subject appeared however to be making a good recovery before sacrifice. Since no signs of injury appeared in the second subject (corresponding to datum point E) no autopsy was done. Both animals were anesthetized during impact and for several hours afterward and no estimate of pain at any time is given. According to Coermann (Ref. 54) a 126 lb sitting Himalayan bear has similar vertical impedance characteristics as a 190 lb seated man. In particular the resonant frequency of the bear is surprisingly very close to that of the man. It is interesting to note that if we choose A_o slightly to the right of point K at 15 g and V_o between points D and E at 47 ft/sec we calculate a natural frequency

$$f_n \cong \frac{1}{2T_o} = \frac{A_o}{2V_o} = 5.1 \text{ sec}^{-1}$$

Coermann (Ref. 54) reports resonances in a vertically shaken seated bear at 4.3 sec^{-1} and in man at 5.0 sec^{-1}, in good agreement with the calculated frequency above.

The great distance in Fig. 13 between points D, E, and R on the one hand and points L through P on the other may stem from one or both of two differences in experimental technique. First, the tests suggesting the lower value of V_o were willingly borne without injury by human subjects and, second, these subjects were completely free of restraint. It is unfortunate Holcomb has not provided more data on subjective reactions

to impact. The subject of datum point L for example felt severe general shock and pain, but it is not clear if the reaction of the subject of point R was similar or different in this regard. In addition to the differences between the parameters a and v, shown in Fig. 13, there was a much higher peak acceleration (95 g) in Swearingen's data at point L than in Holcomb's peak acceleration (45 g) corresponding to point R.

For the purpose of civil defense shelter design we recommend setting A_o = 7 g and V_o = 11 ft/sec for longitudinal impact of seated men.

There does not seem to be good experimental evidence to locate the long duration asymptote (A_o) when force is applied through the feet of men standing with knees locked. Taking V_o = 9 ft/sec and the natural frequency $f_n = 10 \text{ sec}^{-1}$ and using the formula $T_o = (1/2)\,(1/f_n)$ we compute

$$A_o = \frac{V_o}{T_o} = 5.6 \text{ g}$$

which is very close to the suggested asymptote for the seated position.

e. Static and Dynamic Frequencies

Hirsch (Ref. 19) has statically loaded standing men with weights and observed the average downward defelection of iliac crest in three men to be 1500 lb/in, which for a body weight of 160 lb indicates a natural skeletal frequency of 9.6 Hz. This is in good agreement with the observations by Swearingen of the free vibration of the shoulders of men standing on a hard platform as it was dropped onto a rigid base (Ref. 20). The agreement between statically and dynamically measured frequencies further suggests that in the standing position at least skeletal deformation frequency is not influenced to a great extent by a secondary oscillator which contains the viscera attached to the skeleton.

Dieckmann (Ref. 22) reports two maxima in the impedance versus frequency curve for men standing on a vertically, sinusoidally vibrating platform: the larger in the range 4 to 5 Hz and a smaller one near 12 Hz. He defines impedance as the ratio of force exerted by the platform to its upward speed. The same writer also reports the frequency dependence of

the impedance of a sitting man to vertical motion. There is again a strong peak near 5 Hz but no further significant resonance out to 30 Hz. Dieckmann suggests that a sitting man can be treated as a single degree-of-freedom oscillator and that a standing man is essentially the same equivalent oscillator modified by the addition of a secondary mass which has the effect of introducing a secondary resonance at a higher frequency than the first. Since the maximum at 3-5 Hz retains its magnitude and location when the subject sits, this is a reasonable view. Moreover, voluntary tolerance levels reported by Hirsch and Swearingen for standing men do not differ appreciably from their corresponding levels for seated men, i.e., a single "failure mode" is controlling in both positions. Swearingen gives average acceleration (at the platform) $a = 32$ g and speed change $v = 8.3$ ft/sec, plotted as point Q in Fig. 13 slightly below point L. It would be natural to infer that the limiting strain in Hirsch's and Swearingen's standing and sitting subjects was associated with the visceral mode reported by Coermann and others at 3 to 4 Hz and the major impedance peak seen by Dieckmann at nearly the same frequency--despite the fact that the observed skeletal vibration took place at 10 Hz. There is however one difficulty with this view. If the limit V_o for the visceral longitudinal mode was located by Swearingen and Hirsch at $V_o = 10$ ft/sec; if the natural frequency of this mode is indeed 3 to 4 Hz; and if the seated man can be represented by a single degree-of-freedom system, then the corresponding asymptote A_o becomes

$$A_o = \frac{V_o}{T_o}$$

$$\frac{1}{4}\frac{1}{f_n} \le T_o \le \frac{3}{4}\frac{1}{f_n}$$

$$\frac{10 \times 4 \times 3.5}{3 \times 32} = 1.45 \text{ g} \le A_o \le \frac{10 \times 4 \times 3.5}{32} = 4.4 \text{ g}$$

which is regularly exceeded in pilot ejection. Using $f_n = 5$ Hz raises the upper bound on A_o to 6.3 g, still below the regularly observed value. This is an anomaly which will receive further attention later. As has

been implied by the location of the "longitudinal visceral" limit in Fig. 13, skeletal behavior rather than visceral may be responsible for the asymptotic limits A_o = 10 g, V_o = 10 ft/sec.

f. Poor Posture

It is important to note that none of the impacts described in the foregoing was designed to deliberately excite flexural modes of the vertebrae. Hirsch's and Swearingen's subjects presumably held themselves upright as much as possible so that the load was transmitted through the whole vertebral column. Although Stapp's second group of volunteers were subjected to a force not along any body axis, they were very tightly restrained and everything was done to prevent flexure of the back, which was looked upon as highly dangerous. (The head was not restrained however.) In his earlier work Stapp found a 50% and more reduction in tolerable deceleration when the back was deliberately bent to its forward limit before impact (Ref. 14).

Laurell and Nachemson (Ref. 55) studied 55 cases of successful ejection from Swedish aircraft in flight and concluded there was a strong likelihood of spinal injury in one kind of high performance craft unless the subject's posture was correct at the time of ejection. Catapulting without preparation for example led to injury in six cases of seven. Accelerograms are not given but the acceleration pulse experienced by the injured fliers seems to have been of the type represented by points J and K in Fig. 13.

Another kind of Swedish craft tended to be much more tolerant of poor posture. Significantly, the ejection system aboard this plane leads to peak accelerations near 15 g as against 20-25 g in the system which caused the injuries. Aircraft ejector accelerograms reported by Latham (Ref. 15) are triangular, peak near 16 g, and last about 160 msec (datum point U, Fig. 13); Latham mentions no injuries with his system. However, subjects of all these ejections are probably athletic young men and all were harnessed to some extent.

The bowed back may correspond to a completely different equivalent oscillator from that for a straight back and the long duration

asymptote, A_o, for voluntary tolerance of unrestrained or unprepared men may lie in the region between 2 to 5 g and natural frequency f_n, close to or within the range 1 to 3 Hz noted by Dieckmann in horizontal whole body flexure.

g. Influence of Age

At this point we must look upon both age and posture as potentially important but largely unexplored factors in impact tolerance.

The influence of age is suggested by the data collected by Stech and Payne (Ref. 29) who extrapolate to find that at 119 years the human vertebrae would break in 50% of the individuals under normal gravity (1 g). In fact age is important in all kinds of injury as suggested by data assembled by the Automotive Crash Injury Research group of Cornell University establishing a clear trend for increased likelihood of serious injury with advancing age in automobile crashes (Refs. 50 and 56). Under comparable conditions injuries were fatal about twice as often in a group of victims over 60 years of age as in a group in the range 20-59 years (Ref. 57).

In the absence of a wider spectrum of data we must face the possibility of impact injury in shelters even when conditions fall within the smallest of the "safe" zones in Fig. 13.

B. Whole Body Harmonic Behavior

1. Summary of Tolerances

The data described in the foregoing may be summarized by listing three body resonances and their characteristics, as has been done in Table III. The last column contains the maximum tolerable relative displacements on the simple model of the undamped spring and each value is found from the formula:

$$x_o = \frac{V_o}{2\pi f_n} \tag{7}$$

Table III

CHARACTERISTICS OF THREE BODY RESONANCES

No.	Direction of Motion	Frequency (Hz)	Description	A_o (g)	V_o (ft/sec)	x_o (in.)
1	longitudinal	3.5	Abdomen/thorax	10?	45?	24?
2	longitudinal	10	Musculo/skeletal	7.5	11	2.1
3	transverse (head-on and side-on)	2	Visceral	17	80	75

and are in addition to any displacements due to normal gravity. The frequency listed for resonance 3 in Table III does not agree with the findings of Clark, Lange, and Coermann (Ref. 58) who shook semisupine men up and down; they report a peak strain per unit acceleration between 6 and 8 Hz. They did not explore below 2 Hz. The existence of a resonance does not, of course, imply the corresponding strain is physiologically important in determining motion tolerance. [There must of course be many other vibrational "resonances" of the human body beyond those listed in Table II. See, for example, Latham (Ref. 15). However, except for the head-shoulder subsystem which will be discussed later, the "subsystems" listed above seem to make it possible to treat all existing motion tolerance data in an organized way.]

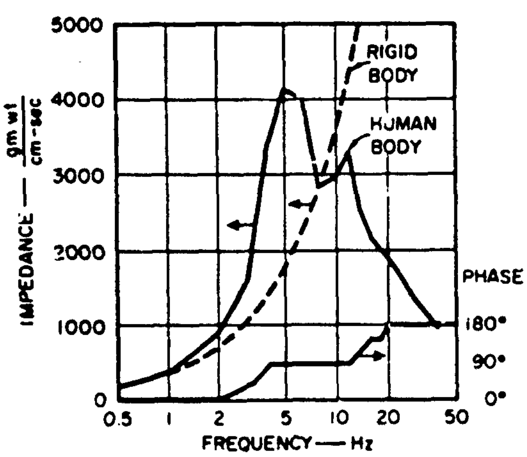
The physiological meaning of the values of x_o in Table III above is not clear. Certainly the entry for resonance No. 3 cannot refer to any actual distortion of a linear elastic element in the human body. Presumably nonlinear and viscous effects predominate long before the full distortion of 75 inches is reached. The great disparity in magnitudes between x_o for entry 3 on the one hand and the values for entries 1 and 2 on the other will be interpreted simply as a measure of the difference between the hardihood of the human structure to transverse and to longitudinal impact. Lombard, Close, Thiede, and Larmie (Ref. 59) impacting guinea pigs on sleds find a markedly greater degree of freedom from lethal injury in transverse contrasted to longitudinal impact.

There is independent evidence in the work of Coermann and others (Ref. 23) for the validity of the value of x_o in row 1. They measured maximum outward abdomen wall displacements at resonance to be 6.75 cm/g (Ref. 23), which at 7.1 g would imply a peak displacement of the order of 19 inches. This was not a movement parallel to the body's longitudinal axis but resulted from such a movement, and the similarity in magnitude to the calculated value is striking. (Again, the simple model would probably become inapplicable before a displacement this large was reached.)

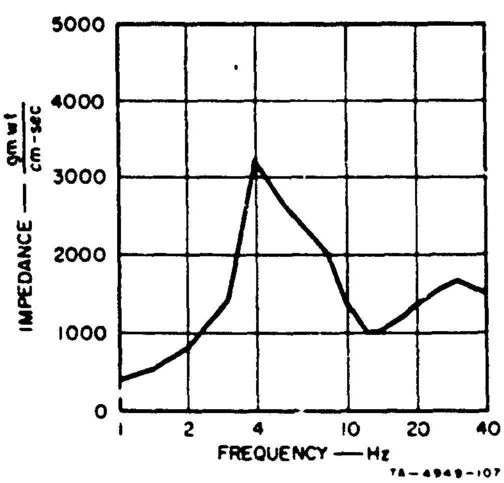
In Coermann's work the subject was lying on the horizontal moving table and motion of the whole skeleton was deliberately prevented by the method of attachment.

2. Impedance Characteristics

A great deal of light is shed upon and perhaps some corroboration is given to the views expressed by rows 1 and 2 of Table III by the work of Dieckmann (Ref. 10) who vibrated sitting and standing men vertically at many frequencies and reported magnitude and phase of mechanical impedance (ratio of sinusoidal components of force and speed at the platform). (See Appendix D for discussion of impedance.) Some of his results are reproduced in Figs. 14(a) and (b). For both postures there is a large broad peak



(a) Standing



(b) Sitting

FIG. 14 FREQUENCY DEPENDENCE OF WHOLE BODY IMPEDANCE (from Ref. 10)

near 4-5 Hz and one lesser peak at higher frequencies. Were the 160 lb man a rigid body, his impedance would depend on frequency in the way shown by the dashed line in Fig. 14(a). Since at 40 Hz the magnitude of the actual impedance has fallen far below that of the rigid body of the same mass, the resonances at 4-5 Hz (both standing and sitting), at 10-12 Hz (standing) and at 20-30 Hz (sitting) must involve the major part of the whole body weight, if not all; that is, at frequencies as high as 40 Hz probably the whole body is supported on "soft" springs and the resonances of these spring systems lie below 40 Hz. Despite the closeness of the resonant frequencies found by Dieckmann for the whole body and by Coermann and others (Ref. 23) for the abdomen-thorax they cannot be attributed to exactly the same single physical system because the total weight of the human viscera is not a large enough fraction of the whole body weight.*

3. Body Models

Human impedance characteristics may be duplicated fairly well with two or more coupled harmonic systems using lumped mechanical constants. Dieckmann (Ref. 22) suggests the schemes shown in Figs. 15(a) and 15(b) to account for his observations. The impedance and its phase angle have been calculated and are plotted in Fig. 16 for the representation of

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* Weights in grams of several larger visceral bodies are given by Ref. 62 as follows: lungs, 950; heart, 312; liver, 1500; intestines, 1600; stomach, 129; kidneys, 313. These total 4800 gm or about 11 lb to which may be added the mass of connective tissue and visceral contents, making up something less than 1/10 whole body mass. Thus it seems unlikely that the actual oscillating system behind the observed body resonance at 4-5 Hz is the viscera moving with respect to the rest of the body. Such a visceral mode can clearly be injurious. Since Coermann and his co-workers did not restrain expansion and contraction of the rib cage and its attached flesh, the actual mode they studied was an oscillation involving the chest wall and the abdominal covering as well as the enclosed viscera. Motion of chest and contents together might still lead to internal injury if the wall moved more transversely than longitudinally and the viscera moved more longitudinally than transversely. However, adding the whole weight of the ribs (330 g) and sternum (32.5 g) and doubling this value to account for muscle and other flesh brings the total mass of the postulated harmonic subsystem to less than 15 lb.

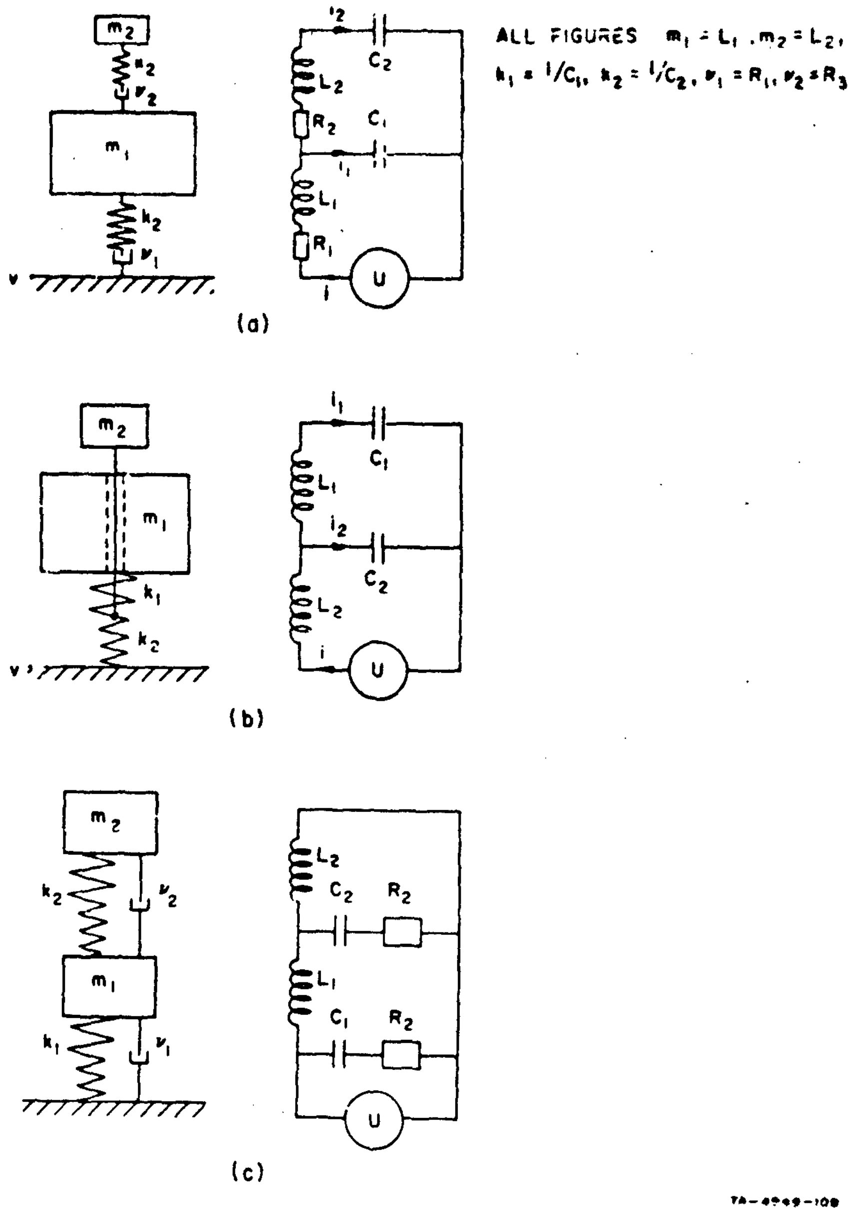

FIG. 15 MECHANICAL AND ELECTRICAL ANALOGS OF THE HUMAN BODY

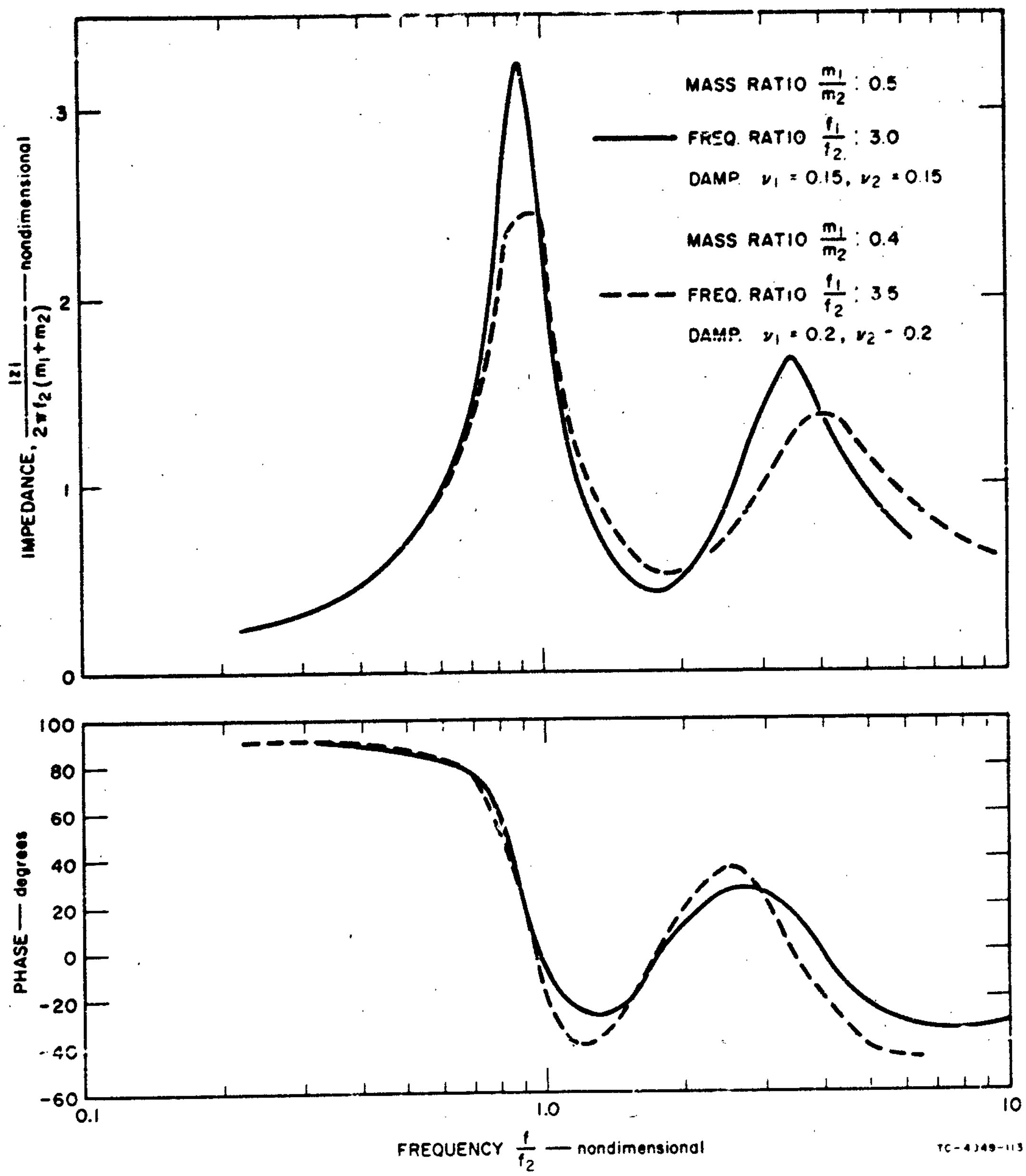

FIG. 16 FREQUENCY DEPENDENCE OF IMPEDANCE ON A TWO-DEGREE-OF-FREEDOM TANDEM COUPLED SYSTEM

Fig. 15(c). Two different mass ratios m_1/m_2 and frequency ratios f_1/f_2 have been used in the calculations. The system touching the platform is considered primary and its parameters bear the subscript "1." Frequency f_1 is defined as

$$\frac{1}{2\pi}\left(\frac{k_1}{M_1}\right)^{\frac{1}{2}} = \frac{1}{2\pi}\left(\frac{1}{L_1 C_1}\right)^{\frac{1}{2}} \quad ;$$

damping is expressed as fraction of critical, viz., damping ν_1 in the primary circuit equals $\pi R_1 C_1 f_1$. These exploratory calculations, while not exhaustive, point to the fact that the frequency dependence is more realistic if $m_1 < m_2$, in agreement with Dieckmann's scheme in Fig. 15b (where he has labelled the primary mass m_2). Dieckmann appears to locate the juncture of the two springs at the pelvis; the mass m_2 contains the vertebral column and head. He does not mention visceral movements although clearly these bodies are set in motion (Refs. 23, 53, 61, and 62). Definitive work mapping the responses of various human body regions does not appear to have been done.

a. Sensitivity Curves for Body Models

Tolerance or sensitivity curves or asymptotes, such as those shown in Fig. 1, can be computed for an undamped two degree-of-freedom tandem harmonic oscillator, and some curves of this kind are reproduced in Figs. 2 and 3 for primary and secondary systems, respectively. The input acceleration pulse is rectangular and is applied to the base of the primary system. Ordinates are speed changes in the base; abscissas are average accelerations in the base. The unit of speed change is different for the primary and secondary systems and equals the magnitude of that instantaneous change applied to the single oscillator alone which produces the maximum tolerable spring strain. For example, the sudden change v_1 in the mass (or base) speed of the secondary system (uncoupled to the primary oscillator) may produce peak strain x_o in the spring. If a rectangular pulse lasting twice as long as one natural period of the secondary oscillator and containing a speed change 5 v_1 is applied to the base of the compound system and produces the strain x_o in the secondary oscillator,

then the point abscissa = 5/2, ordinate = 5, appears on the tolerance curve. On both Figs. 2 and 3, the tolerance curve for the single uncoupled oscillator is also plotted for comparison.

Generally in Figs 2 and 3 two ranges of values for the ratio m_1/m_2 have been chosen, one to simulate the dual system pictured in Fig. 15b and another to study the behavior of the viscera attached to the bodily frame. In the first m_1/m_2 falls in the range 1/3 to 2/3; in the second m_1/m_2 is 10 to 20 since the frame carrying most of the weight is looked upon as the primary system. Frequency ratios in the first range are consistent with those used in Fig. 16 and in the second range the frequency ratio is fixed at f_1/f_2 = 5/3.5.

In both series of cases the figures show that generally the tolerance curves for individual oscillators composing compound systems keep the shapes they have when the single oscillator is alone. When the primary mass is much larger than the secondary, the tolerance of the primary oscillator is hardly affected by the presence of the system coupled to it, but the V_o asymptote for the secondary system is considerably below the line, ordinate = 1, and the A_o asymptote, as well, is often lower than the corresponding limit for the uncoupled oscillator. When the masses and frequencies of two coupled systems are close together, the tolerance of each system is reduced below what it would be if the same acceleration pulse were applied to the uncoupled oscillator alone. However, in the cases considered the value of the asymptote V_o never falls below about one-fourth its peak.

The entries in Table III are derived from actual observations on the compound system, except the frequency in row 1 which pertains to an uncoupled subsystem. The sensitivity curves in Figs. 3(e) and (f) for those cases when f_1 = 1.43f_2 show differences between the uncoupled frequencies and asymptotes, V_o, and the coupled parameters in the secondary subsystem which presumably corresponds to the visceral mode at f_n = 3.5 Hz. In fact Fig. 3 suggests that, when alone, this subsystem has a value V_o about four times as large as its value when coupled. The effect of coupling on the frequency is not clear from Fig. 3 but the modal frequency

should lie below the uncoupled secondary frequency when $f_1 > f_2$. Thus to calculate a value of x_o corresponding to an uncoupled visceral subsystem from the equation $x_o = V_o/2\pi f_n$ the observed value of V_o should be multiplied by a number at least as great as 4. In other words, to preserve the agreement between the observed abdominal displacements by Coermann and others (Ref. 23) and the entry for x_o in row 1 the limit V_o for the longitudinal visceral failure mode must be set at

$$V_o = \frac{45}{4} = 11 \text{ ft/sec}$$

instead of 45 ft/sec as shown in Fig. 13. The difficulty with this view, as has been noted before, is that A_o for the compound system is reduced below known limits in longitudinal motion. The question remains unresolved.

Judging from the sensitivity curves in Figs. 2 and 3(a), (b), (c), and (d) corresponding to $(3/2)m_1 \le m_2 \le m_1$, the value of x_o in row 2 should perhaps be multiplied by a factor between 2 and 3, if x_o is to represent a physical displacement in either a secondary or primary subsystem.

(Mathematical derivation of tolerance curves for a two degree of freedom coupled system is carried out in Appendix B.)

We have in longitudinal vibration not only the compound system suggested by rows 1 and 2 of Table III but a further compoundedness in row 2 alone, as indicated by Dieckmann's evidence for two impedance maxima. There may, of course, be more; furthermore, it cannot be inferred that the limiting asymptotes of the conservative envelope (illustrated schematically in Fig. 4) are associated with the simple subsystems uncovered so far. However, the observed frequencies f_n of both the maxima fall fairly well within the range allowed by our asymptotes, A_o and V_o, for row 2; that is

$$\frac{1}{4} T_n \le \frac{V_o}{A_o} \le \frac{3}{4} T_n$$

or

$$f_n \geq \frac{7.5 \times 32}{4 \times 11} = 5.5 \text{ Hz}$$

$$f_n \leq 3 \times 5.5 = 16 \text{ Hz}$$

which suggest a conservative envelope may be associated with the resonances found by Dieckmann; that is, the data plotted in Fig. 13 are not fine enough to permit resolution of two different failure modes corresponding to row 2, Table III.

b. Correlation of Maximum Strain from Steady Oscillation with Impact Data

Dieckmann (Ref. 10) also gives some support to the value of x_o in row 2 by correlating, "degree of tolerance" of sitting and standing men on a vertically shaking table to platform displacement maximum excursion (1 cm or less) and frequency of vibration. Specifically he calculates a number "K" whose magnitude predicts the subjective reaction to the vibration. When K = 100 the motion is described as "extremely unpleasant, to be considered for at most 1 minute exposure." For K greater than 100 the vibration is "unbearable." Furthermore, he states the relation between K, maximum table displacement, s (in mm), and frequency, f, as $K = sf^2$ in the range 0-5 Hz, $K = 5sf$ for $5 \leq f \leq 40$ Hz and $K = 200\ s$ for $40 \leq f \leq 100$ Hz. On the simple oscillator model* maximum distortion x_o of the spring is from the definition of impedance, including viscosity,

$$x_o = \frac{2\pi f\ s|z|}{k(1 + 4\nu^2)^{1/2}} \tag{8}$$

where

k = spring constant

f = spring frequency

* dash-pot and spring in parallel.

s = maximum displacement of base or platform

z = impedance or ratio of peak force on base to peak speed of base

ν = fraction of critical damping

From the magnitude of the observed impedance maximum, viz., 4×10^6 dyne cm^{-1}, and the value of table displacement corresponding to $K = 100$ and to frequency = 5 Hz, viz., 0.4 cm, the maximum tolerable spring distortion x_o equals 0.4 inch provided the suspended mass is taken as 60,000 gm (130 lb) and $\nu = 0$. The ratio under steady sinusoidal oscillation between peak table displacement, 0.4 cm, and peak mass displacement, 1 cm, calculated on the simple model from Dieckmann's observations indicates the presence of about 20% critical damping in the simple oscillator equivalent to the human frame (Ref. 5, page 219). Thus to a first approximation $\nu = 0.2$ and from Eq. (8) above, $x_o = 0.4/1.07 = 0.37$ inch. This is six times smaller than the limit shown in Table III for the undamped oscillator. Both viscous effects and "sensitization" would tend to reduce this discrepancy. That is, use of an analog of Eq. (7) which took into account viscosity would result in a lower value of peak strain x_o calculated from the asymptote V_o; furthermore it may be reasonable to assume that body elements become sensitized with repeated strain under steady vibration and are thus somewhat better able to bear a single large strain under impact than a repeated small strain under vibration. A value $K = 1000$ might be a more pertinent limit for one sided impact pulses.

These considerations justify a conclusion of agreement within an order of magnitude between our criterion for tolerable strain in the "longitudinal skeletal" mode, given by the entry in the last column of row 2, Table III, and Dieckmann's criterion for tolerance to longitudinal vibration. (The "coupling" correction does not change the conclusion, since it would be applied to both the entry in Table III and the result of Eq. (8) above.)

It may not be assumed from Table III that standing or sitting men can withstand a static load that compresses any part or all of his body 2 inches. We have suggested with the help of the single oscillator

model that the most compression suffered by any of the test subjects was closer to 0.37 inch than 2 inches. Hirsch's measurements (Ref. 19) indicate a static load of 1500 $\times$ 0.37 = 550 lb is needed to depress the iliac crest 0.4 inch. Such a load is not ordinarily borne by untrained men. However, the torso very clearly cannot be equated to a single oscillator; at least two oscillators must be used to reproduce even the gross behavior and the proportion of the total bodily strain revealed by the movement of the iliac crest is not known.

The similarity between the shoulder oscillation frequency seen by Swearingen and others (Ref. 20) under impulsive impact, viz., 10 Hz, and the frequency at the lesser impedance peak reported by Dieckmann, viz., 10 to 12 Hz, makes it likely that the shoulders and arms may be parts of one of the two major vibrating masses postulated by Dieckmann. If a sitting subject held his hands in his lap, this resonance might disappear or be altered in frequency. Dieckmann does not report the position of his subject's hands but his work shows a substantial shift in the lesser impedance peak resulting from sitting.

c. Anatomical Mapping of Resonances

With knowledge available now we can only speculate on the anatomical meaning of the several bodily resonances and sensitivities. Figure 17 suggests the possible sites in the body which may be critical for its motion response. In standing men (Fig. 17) the resonance at 4 Hz seems to involve the whole body down t nd including the hips. Although apparently distributed throughout the sk..eton the "spring" must be found largely between hips and feet in the standing man. The persistence of the resonance at 4 Hz when the man sits (Fig. 17) shows that the torso itself can be excited through the back at the same frequency. Thus, in Fig. 18 spring constant k_1 and total body mass $m_1 + m_2 + m_3 + m_4 = M$ may be related by

$$\frac{k_1}{M} \cong 4\pi^2\, 16\ \mathrm{sec}^{-2}$$

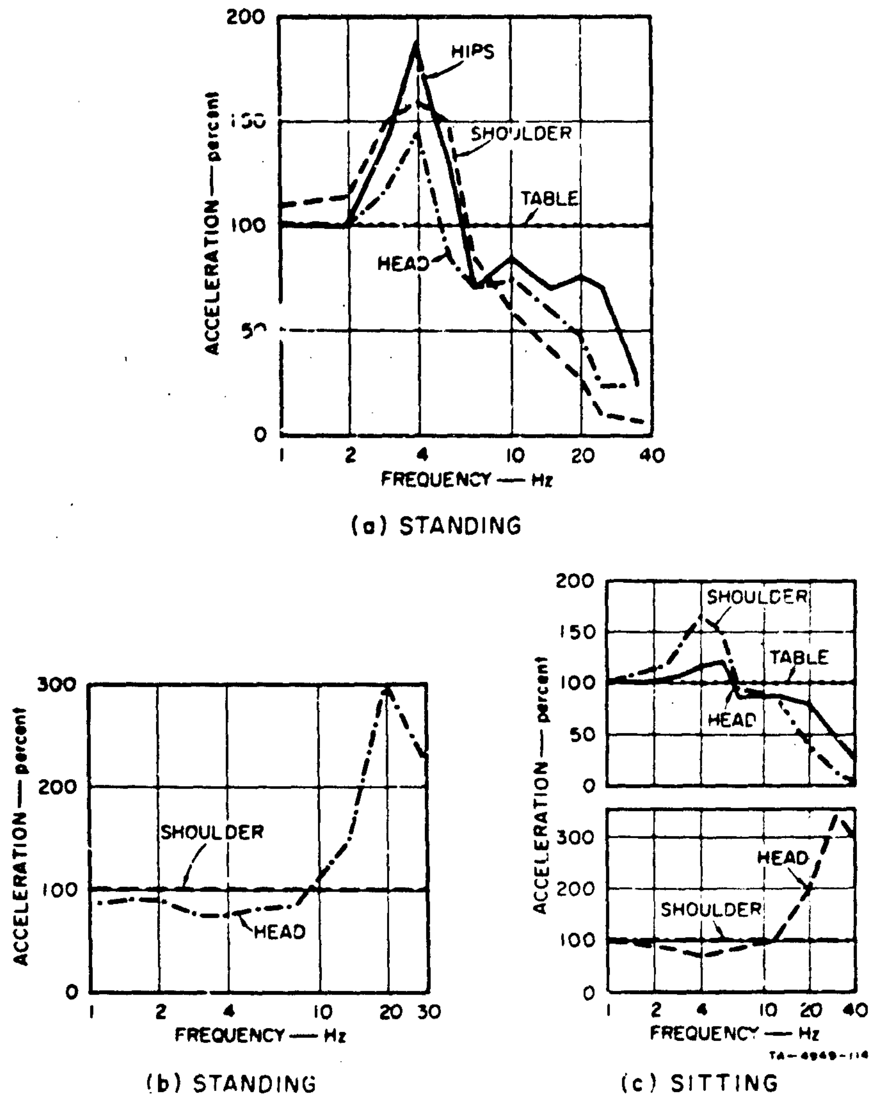

FIG. 17 TRANSMISSION OF STEADY SINUSOIDAL MOTION THROUGH STANDING AND SEATED MEN (from Ref. 10)

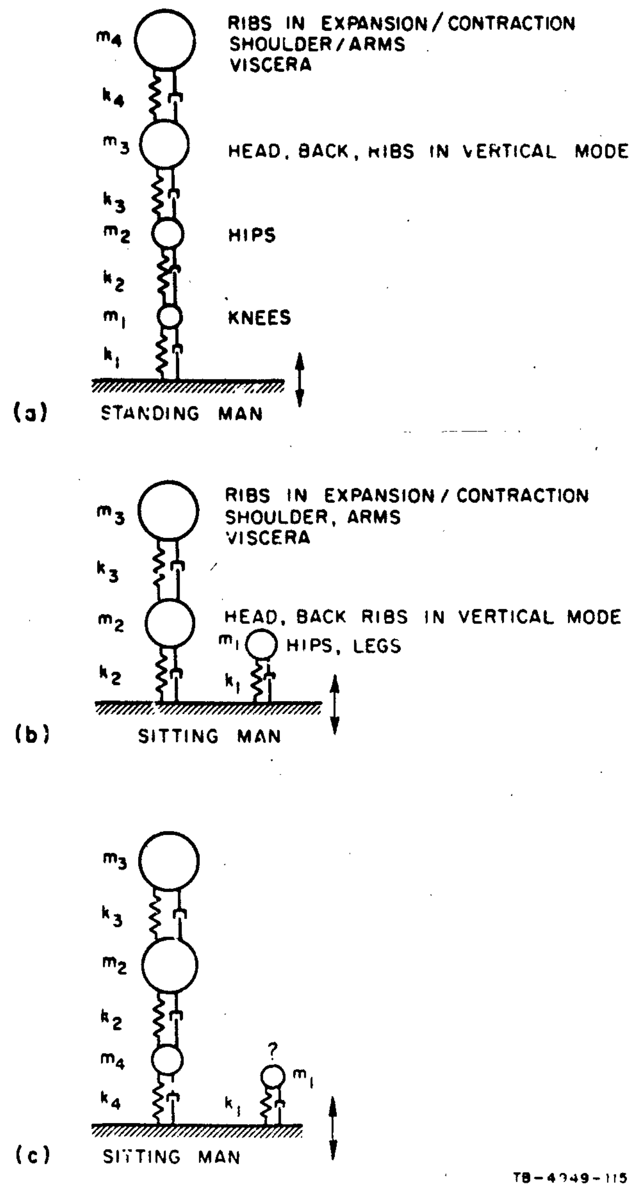

FIG. 18 SUGGESTED SIMPLIFIED MECHANICAL ANALOGS OF THE HUMAN BODY

and

$$\frac{k_4}{m_4} \cong 4\pi^2\ 16\ \text{sec}^{-2}$$

Subsystems containing k_2, k_3, m_2, and m_3 are still "rigid" at this low frequency. At 10 Hz both the springs represented by constant k_1 and k_4 are "soft" and we have the very approximate relation

$$\frac{k_1 k_2}{(k_1 + k_2)(m_2 + m_3)} \cong 4\pi^2\ 100\ \text{sec}^{-2}$$

Finally at 20 Hz the system involving k_3 and m_3 goes through resonance:

$$\frac{k_3}{m_3} \cong 4\pi^2\ 400\ \text{sec}^{-2}$$

but because of the isolation given by lower springs this resonance has no influence on the impedance measured at the platform. It shows up only in the relative displacement of head and shoulders.

The systems shown in Figs. 18(b) and (c) represent the sitting man. The legs and feet, which make up about 1/3 of the total body mass (Ref. 63) must be spring connected to the chair since at 20 Hz such a mass acting as a dead weight should contribute about 2.4×10^6 dyne cm^{-1} sec to the total impedance, but the observed impedance in this frequency range is less than that (Fig. 14(b). Therefore a part of the hips-legs subsystem in the sitting position is very likely in series or parallel with the other two systems as shown in Fig. 18(b) and (c), and this added element in combination with the weight of the back must have a resonant frequency below 10 Hz. If such an element is present, it very likely has an important role in reducing the importance of the back resonance (k_2).

Although the existing impedance and transmissivity data (Figs. 14 and 17) deserve more sophisticated treatment than given above, the foregoing does suggest that the major body components are articulated in such a way that their sensitivities to strain are adequately explored by the

data of Fig. 13 extending over durations 6 to 250 msec. Asymptotes V_o for the major bodily subsystems and failure modes would seem to be firmly established. The possibility of the existence of limits connected with minor bodily subsystems is discussed later.

C. Other Modes of Bodily Response

There are other bodily resonances, of course. Human visual acuity is sometimes reduced at frequencies between 40 and 100 Hz presumably because of an eyeball resonance (Ref. 26). Damage by such a mechanism has not been explored. Skull resonances are found between 300 and 400 Hz. A hand-arm resonance near 35 Hz affects handholding of a vibrating rail (Ref. 64). None of these is likely to be important to shelter occupants; however, reaction of the whole body to horizontal motion may be.

1. Transverse Flexure

Dieckmann has surprising success with the view that human tissue has on the average a characteristic spring constant which he calculates from the mass and natural frequency (4 Hz) of the whole body under vertical vibration (Refs. 22, 25, and 64). The spring constant then gives a relation between force and strain, that is, a value of Young's modulus. Treating the whole body as a uniform bar of certain length, diameter and mass, he easily calculates its reaction to horizontal motion applied to a platform on which a man stands. The frequency for whole body, flexural or bar-like oscillation of standing men under horizontal platform motion turns out to be 2.8 Hz. Dieckmann (Ref. 25) argues that his observations show a complete standing wave along the body length at frequencies between 2 and 3 Hz, thus confirming the validity of his approximate calculations. By assuming a maximum tolerable horizontal displacement between head and feet of 1 ft, we can calculate tolerance limits (A_o and V_o) for "half-wavelength" bending of the whole body (resonant frequency presumably equal to 2.8/2 Hz) as follows:

$$x_o = 1 \text{ ft}$$

$$V_o = 2\pi x_o f_n = 2\pi\, 1.4 = 8.8 \text{ ft/sec} \tag{9}$$

$$A_o \cong 2 f_n V_o = 0.8 \text{ g} \tag{10}$$

There are two modes of possible injury: direct strain of body tissues during bending and the secondary effects resulting from toppling. As for the first mode, Dieckmann's criteria (Ref. 25) imply that platform displacement above 1 inch at 1.4 Hz corresponds to $K > 100$. For $K = 1000$ the table amplitude becomes about 10 inches. Dieckmann provides no measurements on which to estimate amplification. These standards assume continuous oscillation and are for single impacts conservative, as we have seen. For our purposes we can probably regard the tolerance limit as closer to 10 inches than one. Results of tests in both the sagittal (fore and aft) and frontal planes showed no appreciable difference. As for the second possible injury mode, the center of mass of the human body is quite near the hips midway from feet to head (Ref. 63) so a relative displacement of one foot between feet and head in the fundamental mode implies an approximately 6-inch displacement of the center of mass from a vertical line through the feet. This would seemingly be the least needed to overcome usual spreading of the feet and to topple the person. Presumably all of Dieckmann's subjects were able to remain standing during testing on the horizontally shaking platform; however, it appears that maximum platform acceleration was never more than 1/5 g and probably the subjects were allowed to brace themselves by positioning their feet apart. Very likely the limits estimated above corresponding to whole body horizontal flexure will be approximately correct for both injury modes in horizontal vibration. Likelihood of injury actually resulting from toppling is discussed later. The above asymptotes have been entered in Fig. 13, marked "horizontal flexure."

An earlier worker, von Bekesy, (Ref. 65) reports resonances in horizontally oscillated sitting and standing men at frequencies of 1.6 and 0.6 Hz, respectively; but von Bekesy's work does not seem to distinguish clearly between a whole body resonance and one peculiar to the head and shoulders alone. In fact his measurement of 1.6 Hz may easily be associated with just such a limited movement. Some of Dieckmann's data confirm this partly by showing an amplification of head motion contrasted to shoulder motion near a frequency of 1 Hz, which incidentally seems to disappear before 1.5 Hz is reached.

2. Whiplash Head Motion

The relative transverse motion of head and shoulders is responsible for the injury known as whiplash and may constitute a threat, particularly to seated people in a shelter. Presumably the maximum allowable excursion would be less than 1 ft; consequently, since the natural frequency is also about 1.4 Hz (Ref. 25), the asymptotic limits V_o and A_o would be somewhat lower than those calculated just above [Eqs. (3) and (10)] for the half-wavelength bending of the whole body, viz.,

$$x_o < 1 \text{ ft} \tag{11}$$

$$V_o < 8.8 \text{ ft/sec} \tag{12}$$

and

$$A_o < 0.8 \text{ g} \tag{13}$$

It should be noted that 8.8 ft/sec = 6 mi/hr and that since the major effect of rear end auto collision is over in a period of less than 1/2 sec, the limits calculated above for the threshold of whiplash injury would seem highly conservative. However, it will be seen below that if the production of pain is taken as limiting the relative motion of head and neck the foregoing estimates have experimental justification.

The voluntary tolerance levels for blows to the padded head found by Lombard, Ames, Roth, and Rosenfeld (Ref. 24) probably should be understandable on the simple model implied by the asymptotes calculated in Eqs. (6) and (7) since the subjectively described limits to further exposure often involved neck pain and never anything like brain malfunction or skull injury. Blows were delivered by a steel hammer 9.4 or 13 lbs in weight to top, front, side, and back of the helmeted head. Accelerometers were attached to the hammer. Although accelerograms were not reproduced in the report the acceleration histories appear to have been generally trapezoidal.

In these experiments the impact was delivered to the mass of the linear oscillator instead of to the platform, but the difference is not

important to the applicability of our simple model. The striking mass and the mass of the head were nearly equal and the transfer of momentum took place relatively quickly so that the equating of kinetic energy in the striking mass to potential energy at maximum spring distortion (neglecting, as before, viscous losses during head motion) is still valid, i.e., $x_o = V_o/\omega$. If energy loss in the padding is also negligible then V_o equals speed of striking object.*

Neck pain--as often ligamentary as vertebral--was an important element of subjective response to the frontal blows observed by Lombard and fellow workers, and appears to have limited impact speeds to below 5 ft/sec, which compares well with the limit stated in Eq. (12) above. Thus, for the whiplash response

$$A_o = \pi f_n V_o = \pi 1.5 \times \frac{5}{32} = 0.73\ g \quad \text{and} \quad x_o = \frac{V_o}{\omega} = 6.4 \text{ in.} \ .$$

Observational data do not seem to be available to determine whether such a value for A_o is a realistic long duration tolerance limit for the nodding head. Although centrifuge studies have been directed toward finding whole body transverse acceleration tolerances, their results often show limits below 5 g. Specific tests with the head unsupported have not been found.

Only one of Lombard's subjects who were struck on the back of the head (as contrasted to those receiving frontal blows) stopped the escalation of impact energy because of neck pain. His tolerance limit was very similar to that suggested above for frontal blows.

The excursion limit x_o calculated above for fore and aft motion of the head relative to the shoulders is anatomically reasonable.

The asymptotes computed above have been entered in Fig. 13 and marked "whiplash."

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* Padding in many helmets serves mainly to distribute force over wide area of the head and not to absorb a large part of the striking object's kinetic energy. Absorption of energy reduces actual V_o below impact speed.

3. Vertical Head Motion

For blows delivered to the top of the head of a seated subject voluntary tolerance ceased due to vertebral neck pain at speed changes of 5.7 and 6.1 ft/sec and durations approximately 8.5 and 13.2 msec, respectively (Ref. 24). The model pertinent to this mode of impact is that pictured in Fig. 18(b) and (c). The downward blow is delivered to m_2 which rests atop the spring k_2. If m_4 in Fig. 18(c) is large enough or spring k_4 bottoms out early enough, then the reaction of the head or m_2 will be that of a linear system of natural frequency near 20 Hz only slightly influenced by the soft spring k_3 and its mass m_3, which will not move appreciably during the short lasting impact. Since durations of 8 to 14 msec are considerably shorter than a natural period of the principal system, i.e., 50 msec, the short duration asymptote for overstrain of the equivalent linear oscillator must lie near V_o = 6 ft/sec and the long duration asymptote falls near

$$A_o \cong \pi f_n V_o = 12 \text{ g}$$

The value of x_o corresponding to these data is $x_o = V_o/2\pi f_n$ = 0.4 in.

These values of maximum tolerable relative displacement x_o = 0.4 inch and average acceleration A_o = 12 g turn out to agree almost exactly with the values of peak displacement and peak acceleration of the platform corresponding to a tolerance constant K = 1000, as given by Dieckmann (Ref. 22) for a vertical excitation frequency of 20 Hz. It is also true that in this experiment Dieckmann found the head moving only 25% to 50% less than the platform in the frequency range 20-30 Hz [Fig. 17(c)]. Thus, if the foregoing calculation of displacement of the head under downward impact is correct, two quite dissimilar series of experiments define the limit of voluntary tolerance to vertical motion in terms of nearly the same head displacement. Dieckmann however does not explicitly locate the anatomical site of the pain forbidding stronger vibration of seated men in the range 20 to 30 Hz.

Asymptote A_o = 12 g, V_o = 6 ft/sec are marked in Fig. 13 as "vertical head acceleration." The impulsive asymptote V_o for vertebral longitudinal motion applied through feet or buttocks lies above 6 ft/sec presumably because of the isolating effect of intervening body tissues.

4. Possible Unexplored Hazards

Because the empirical data summarized in Fig. 13 are limited to a restricted region along the time or duration axis between 6 and 250 msec, there is always the chance a minor* resonance with a half-period falling outside this region will set tolerance limits not suggested by a conservative envelope drawn through data so far revealed (Fig. 19). With the present knowledge of the response of particular body parts to motion such possibilities can be explored only empirically.

Although in the discussion above neck strain is treated as a kind of by-product of strain in other bodily subsystems, the head-neck could conceivably be such a limiting minor subsystem in its own right since Dieckmann does not report unusual neck strain at 20-30 Hz [Figs. 17(b) and 17(c)]. (Phase observations also point to a possible local resonance in the motion of the head with respect to the shoulder, Ref. 10.) However, the lesser peak in the magnitude of impedance Dieckmann reports for sitting men in the same frequency region [Fig. 14(b)] cannot be due solely to motion in the head and neck because the total mass of these bodies is not a large enough fraction of total body weight. (The head weighs about 10 lb.)

Sensitivity curves to rectangular acceleration pulses applied to coupled primary and secondary subsystems of this kind are illustrated by Figs. 5 and 6, where the parameters have been chosen to simulate roughly the head rooted to the torso by the neck (m_1/m_2 = 10 and 20; f_1/f_2 = 1/5). Sensitivity in the primary, which contains most of the system mass, is hardly affected by the coupling; and, generally, the secondary is isolated from the motion by the presence of the primary, but the degree of isolation

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* Minor in the sense of not affecting perceptibly the overall mechanical impedance of the body.

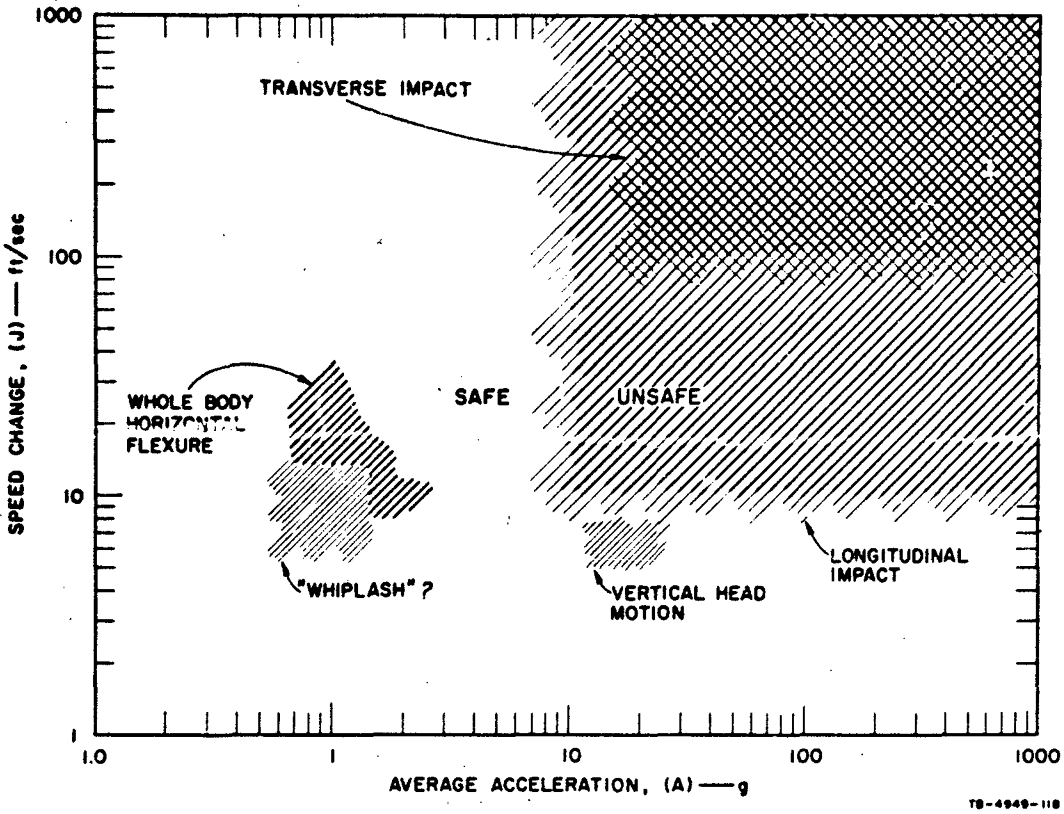

FIG. 19 HUMAN TOLERANCE TO IMPACT

appears to vary considerably with pulse duration. Figure 6 shows a region of heightened sensitivity near the natural period of the secondary.

For both standing and sitting men vertically shaken at 20 to 30 Hz Dieckmann's criteria associate a value of K = 100 with maximum table displacement

$$s = \frac{K}{5f} = \frac{100}{5 \times 25} = 0.8 \text{ mm}$$

The maximum spring distortion implied by Fig. 6 is seven times less than what is calculated by Eq. (7) applied to an undamped single degree-of-freedom oscillator at 25 Hz, viz.,

$$x_o = \frac{1}{7}\frac{V_o}{\omega}$$

If neck distortion is comparable to s then

$$\frac{1}{7}\frac{V_o}{\omega} \cong s = 0.8 \text{ mm}$$

or

$$V_o \cong 0.56 \text{ cm } 2\pi \; 25 \text{ sec}^{-1} \frac{1 \text{ ft}}{30 \text{ cm}} = 2.9 \text{ ft/sec} \qquad (14)$$

Therefore, although Dieckmann does not locate anatomically the limiting strain, by choosing the asymptote $V_o = 11$ ft/sec for the longitudinal mode we may conceivably place the neck in hazard. The contrary conclusion emerges from Swearingen's data (Ref. 20) shown as follows. When he dropped stiff-legged subjects, average shoulder acceleration was 5.8 g, speed change 8 to 9 ft/sec, pulse duration 45 msec, and no neck pain was reported. If we take $A_o = 5.8$ g as the long duration asymptote for the supposed single degree-of-freedom system consisting of neck and head, then

$$V_o \cong \frac{1}{2} A_o T_n = \frac{1}{2} \times 5.8 \times 32 \times \frac{1}{25} = 3.7 \text{ ft/sec} \quad . \qquad (15)$$

This asymptote $V_o = 3.7$ ft/sec is a safe level of motion at the shoulders. But the motion ordinarily reaches the shoulders through a primary system of large mass and low frequency; and from Fig. 6 we can infer that motion applied to the base of the compound system may have an asymptote at least seven times as large, or

$$V_o = 7 \times 3.7 = 25 \text{ ft/sec} \quad .$$

which is above the already established asymptote $V_o = 11$ ft/sec.

One of Stapp's volunteers (Ref. 13) seated at an angle 45° to the upright and braked from 29.2 ft/sec in 97 msec, underwent an average deceleration along the spine of 6.7 g and a speed change of 20 ft/sec in the same direction. Shoulders were tightly strapped to the chair but the head was not externally fastened. (The head was helmeted.) This datum seems to be consistent with the value $A_o = 5.8$ g chosen above. Neck pain (without injury) was sustained by a few of Stapp's volunteers

but such pain has been associated with rotation of the head, not compression or stretching of the neck along the long axis of the body.

If in the computation of V_o from Dieckmann's criteria, K is chosen equal to 1000 which has been suggested earlier to be an appropriate value, then Eqs. (14) and (15) will be in agreement. The conclusion is that probably the minor subsystem consisting of head and neck does not limit man's tolerance in the longitudinal mode when the motion is applied through the feet or buttocks. (However, for a man whose shoulders are constrained to move with the given motion but whose head and vertebral column are free, the limiting asymptote V_o is associated with a relative displacement of head and shoulders, as indicated by the asymptotes marked "vertical head acceleration" in Fig. 13 and discussed earlier. This behavior is probably not connected in any way with a "minor" subsystem but has been explained above as arising in major bodily components of which the head and neck are but small parts.)

In the cases of sitting and standing men, exploration of effects due to input pulses falling along the time axis at points corresponding to duration a great deal less than 20 msec cannot be important except perhaps in the foot because of the excellent high frequency absorption ("soft springs") afforded by the body between the platform and vital organs. At the other end of the time axis, i.e., above 250 msec, the existence of unrevealed sensitivities would seem to require natural periods of the order of 1/2 sec. Aside from the movement of blood under centrifugal pressure, which takes seconds of steady motion, the only likely modes that might be important in this time range are those associated with transverse bending. These have been taken up earlier. The acceleration limit suggested by centrifuge studies (Ref. 11) falls between 10 and 15 g and has been entered in Fig. 13.

We suggest that the data reported in Fig. 13 do indeed cover the important region of the time axis.

D. Brain Injury

1. Models and Theories

It has been suggested that brain concussion* may be the result of displacement of the brain with respect to the skull. Holbourne (Ref. 66) sees the most important mechanism as a rocking or nodding movement of the head attached to the body by an inextensible neck. In his view this leads to swirling or relative rotation of the brain with respect to its case. The rough inner skull surface is then the immediate agency of injury. Peak strain between inner surface of the skull and outer surface of the brain may be a linear function of the maximum relative speed between head and shoulders or equally the maximum angular displacement of the head. In other words, the whiplash harmonic system may be involved and the threshold of concussion injury may be related to its parameters. Unterharnscheidt and Sellier (Ref. 67) use a hydrodynamic model made up of a hard thin shell filled with fluid to explain concussion. They emphasize the contrecoup or appearance of contusions on the brain at a site diametrically opposite the location of the impact and suggest "boiling" within the fluid stemming from negative pressure as the important injury mechanism. The threshold on this model may be related to maximum local skull flexure under the impact or the highest pressure induced in the fluid.

Von Gierke (Ref. 67) like Holbourne suggests that brain damage is connected with a relative movement between brain and skull but points to the brain stem near the odontoid process as the site of injury. He has produced concussion in cats without relative rotation of the head with respect to the shoulders and describes the injurious process as cervical stretching. Clearly this process could also be a feature of the nodding motion emphasized by Holbourne. Since concussion is not produced by slow

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* Steadman's Medical Dictionary, 1966, defines brain concussion as "a clinical syndrome due to mechanical forces characterized by immediate and transient impairment of neural functions such as alteration of consciousness, disturbance of vision and equilibrium." Dorland's Medical Dictionary, 1965, adds to this list of possible symptoms "nausea, weak pulse, and slow respiration."

stretching there must be a dynamic element in von Gierke's theory which may actually make it much like that described below.

Lissner and Gurdjian (Ref. 31) take a hydrodynamic approach to concussion but point to a shear stress in the brain stem area which arises because of a pressure differential in the cranial cavity and the spinal canal. The strain resulting from this stress is the direct cause of injury and the threshold is specified by a value of the product of pressure and time. From studies with four human cadavers the authors estimate a pressure of 30 psi lasting 6 msec within the human skull may lead to concussive symptoms.

2. Observed Threshold

There are quantitative observational data pertinent to primates. Swearingen duplicated in the laboratory certain collisions between automobile dashboards and human heads that had occurred in actual accidents and thus associated a specific injury with an acceleration history (Ref. 30). Since he caused identical dashboards to be struck by simulated heads until the dent produced in the laboratory matched the dent created by the human impact in the car accident, Swearingen assumed the acceleration history of the instrumented dummy matched that of the human head. Observed histories were roughly triangular or occasionally trapezoidal in shape. Among the human injuries there were one death, numerous fractures of facial bones, a few skull fractures, and many brain concussions. Neck injury was undoubtedly of relatively little importance.

Either because the motions of the dummies did not duplicate the actual head motions during the automobile crashes or because the simple model underlying Fig. 13 does not apply, Swearingen's data do not yield clear values for asymptotic limits in the coordinates of Fig. 13. Human and geometrical variability may enter also since Swearingen reports on only nineteen different automobile accidents in each of which the location of impact on the head was somewhat different. Since the nineteen cases were almost equally divided between those with and those without loss of consciousness, the range of the values of average acceleration (a), viz, 20-75 g, and speed change (v), viz. 20-45 ft/sec, found in Swearingen's

investigation must be regarded as near the threshold for brain injury due to longitudinal frontal impact. In Fig. 20 the impacts leading to loss of consciousness are plotted as solid circles; the others as open circles. Coordinates are those of Fig. 13, i.e., speed change and average acceleration. The general facial area of impact is written beside each datum point. Durations of accelerations lay in the range of 10 to 50 msec. Peak deceleration is written in parentheses beside each data point.

Data of Fig. 20 indicate the apparent efficacy of the nose as a cushion in preventing loss of consciousness. In Fig. 20 all blows on the nose alone failed to lead to loss of consciousness, yet the recorded

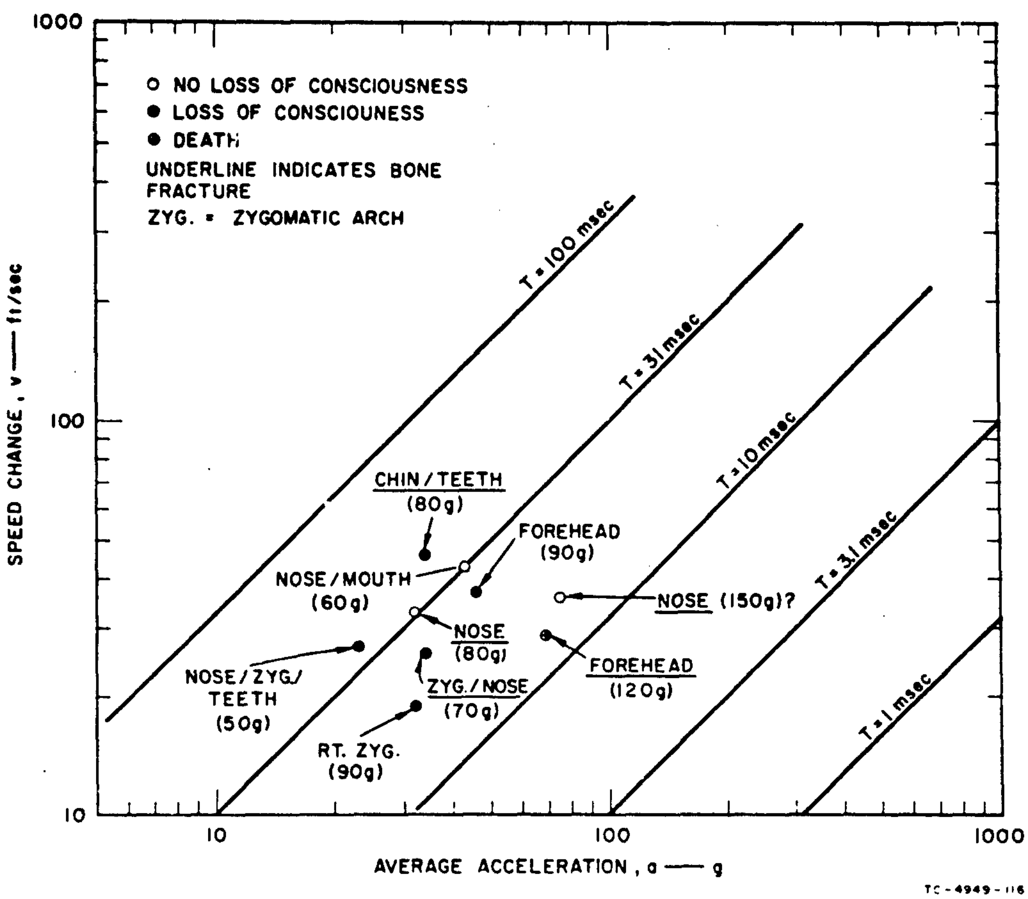


FIG. 20 CHARACTERISTICS OF SOME FACE IMPACTS

parameters, v, a, and peak acceleration, in the corresponding simulated impacts do not differ appreciably from those of simulations of injury-producing blows. This suggests the possibility that the concussion-producing blow contained a much higher average acceleration lasting a shorter time than shown in Fig. 20 and that the peak acceleration or stress in the blow was attenuated before reaching the recorder in the dummy head. Since speed change must be the same everywhere in the head the observed values of v are valid.

Ommaya (Ref. 67) finds blows to the head of monkeys at 22.4 ft/sec lead to concussion in one-third of his trials and three-fourths of the impacts carried out at 33 ft/sec. Site of the blow is not important but the blow must be "square" rather than glancing. Blows were delivered by rubber-tipped pistons and resulted in average acceleration of the head between 10 and 100 g.

The views of Holbourne and von Gierke are for our purposes equivalent. All blows of interest will lead to relative rotation of the head; angular displacement of the brain within the skull and the cervical stretching will very likely be proportional to each other. On both views, then, the whiplash natural frequency, 1.5 Hz, will probably determine that all blows of interest are relatively short-lived and the significant fact in Swearingen's observations will be the speed change, that is, for concussion:

$$V_o \leq 20 \text{ ft/sec} \quad .$$

Data collected by White, Bowen, and Richmond (Ref. 68) put the threshold for human concussion due to a blow from a blunt nonpenetrating object of weight equal to that of the head at

$$V_o = 15 \text{ ft/sec} \quad .$$

This limit has been entered in Fig. 13 along the appropriate part of the time axis and marked as "cerebral concussion." Since the mechanical origin of this disorder is not clearly established, the chance of producing it at lower speed impacts must be kept in mind. Fortunately, most

of the observations underlying the speed limit noted above came from fairly "realistic" or "practical" situations, but the spread of data along the time axis for example is not great (Fig. 20). In the following paragraph another extrapolation of these data is given which differs from the foregoing.

3. Skull Flexure

Franke (Ref. 69) suggests a heavily damped fundamental flexural resonance for the skull at about 300 Hz, which agrees fairly well with a calculated value for an elastic spherical shell with realistic properties. Were such motion responsible for the brain injuries and were skull deflections small enough to be treated as linear functions of force, the two asymptotes, A_o and V_o, should meet near the locus of constant duration equal to 1 msec, and the blows represented in Fig. 20 would be "long lasting." Thus, on this basis, for concussion:

$$A_o < 20 \text{ g} \tag{16}$$

$$V_o = \frac{A_o}{\pi f_n} < \frac{20 \times 32}{\pi 300} = 0.7 \text{ ft/sec} \quad . \tag{17}$$

Such a value of V_o seems much too conservative but there are no data to check it against. Use of a frequency corresponding to a higher mode of oscillation leads to a smaller speed limit. The calculation embodied in Eq. (17) implies that the brain remains still, viz., the frequency of the brain-skull linkage is very low compared to 30 msec, while the skull deforms around it and bruises it. The calculation also assumes that the measured accelerograms refer to the parts of the skull that deform under impact. Swearingen's techniques are not set forth in his report, but it is likely the reported accelerograms record average motion of the whole dummy head.

Franke also reports that group and phase velocities for flexural signals transmitted across living human skulls fall in the range 0.08 to 0.3 mm/μsec. Transit time around a 10-inch head then becomes a matter of 2 to 5 msec. Blows to the head resulting from a fall against a large

heavy object would then appear to be long lasting on a model which looks upon skull oscillation as important and short lasting on models which require gross motion of the whole head.

Swearingen's data in Fig. 20 provide information on bone fracture as well as brain injury. For this purpose when the actual human injury included fracture, the corresponding data point in the figure has been underlined. From the figure alone we would conclude a fracture threshold lay between v equal to 19 and 25 ft/sec.

Fractures are a localized phenomonon associated with high acceleration, short lasting pulses, and will be treated from the shock wave point of view in the following section.

E. Shock Wave Injury

There are some kinds of bodily injury that can perhaps be better understood on a smaller physical scale than that implied in the foregoing discussion of equivalent linear oscillators, and for the smaller scale the concept of shock wave is useful. An abrupt wave of change spreads from a point of impact through a medium causing changes in density and temperature, but, more importantly, bringing the material behind the front into rapid motion. When two or more such waves interfere, the material may be pulled in two directions at once and break. If the material is bone, this is a fracture; flesh is bruised and torn. Blood vessels may be ruptured.

1. Skull Fracture

a. General Considerations

The small-scale, shock wave view-point would seem to apply most directly to skull fracture by impact. This view does have a contribution to make, but as we will see, the shape of the head is probably of equal importance with the shock properties of skull material.

In detail fracture is always a result of unsupportable tension. The simplest kind to imagine is "spall" fracture. Here two planar relief waves moving and pulling in exactly opposite directions meet and cause

the tension to rise suddenly across the plane of collision. Rupture across this plane occurs if the stress is great enough. Tensile stress is aroused also when the free end of an anchored beam is struck. Here the shock wave under the area of impact sets the free end in motion in a direction normal to the beam axis but can only succeed in bending the rest of the beam, putting tension in the convex half and compression in the concave half. The early motion of the struck area depends on the shock characteristics of the material in the striking body and in the beam, but the tension communicated to the rest of the beam depends on beam shape as well as on the early motion of the struck part. Should the beam survive the early motion, it then moves as a single elastic or near elastic body.

Reaction of a spheroidal shell to impact can be discussed in the same terms. At least over a small area spall tensions are possible when a relief wave from the inner free surface meets a relief wave from the free surface of the impacting object or, more likely, from the unimpacted region about the site of the blow. Here the meeting may not be exactly head-on but will very likely be nearly so. If rupture does not occur, then, the tensile wave will pass out of the impact area as bending of the shell takes place. It may be amplified by reflection at free surfaces or by interaction with other tensile waves. At this stage the shell shape controls the peak stress reached. Finally of course the still intact shell may vibrate elastically in any one of several modes or mixtures of modes.

b. Calculation of "Fracture Speed"

The spall tendency can be discussed quantitatively. Goldman and von Gierke (Ref. 26) give the tensile strength of "fresh, compact human bone" as 9.75×10^8 dyne/cm^2 and the acoustic impedance as 6×10^5 dyne sec/cm^3 from which a value of "fracture speed" can be estimated for the human skull, that is, a relative impact speed between skull and a rigid wall great enough to arouse stresses in the skull above the tensile strength. Assuming acoustic impedance approximately equals "shock impedance," we can write peak stress σ in the bone as

$$\sigma = \rho U u$$

where ρU is the shock or acoustic impedance, and u is the impact speed of bone (with respect to rigid wall). Thus if $\sigma \approx \sigma_t$, the tensile strength,* then $u = u_f$, the fracture speed, or

$$u_f = \frac{\sigma_t}{\rho U} = \frac{9.75 \times 10^8}{6 \times 10^5} = 1.6 \times 10^3 \text{ cm/sec}$$

$$= 53 \text{ ft/sec}$$

Impact between bone and a nonrigid material would, in order to cause fracture, have to be at a higher relative speed than 53 ft/sec. In fact if I_b is the impedance of bone and I the impedance of the striking material, then the fraction P/P_o of the rigid wall stress induced at the interface becomes

$$\frac{P}{P_o} = \frac{1}{1 + \frac{I_b}{I}}$$

Values of I and P/P_o are given in Table IV for five different materials For example, the same interface stress is produced by a 10 ft/sec impact

Table IV

REDUCTION OF STRIKING PRESSURE IN SOFT MATERIALS

Material	Shock Impedance (10^6 dyne sec/cm^3)	P/P_o
Iron	4.0	0.87
Granite	1.8	0.75
Aluminum	1.6	0.73
Lucite	0.61	0.50
Wood (ash)	0.06-0.2	0.1-0.25

* In principle, tension is found at the meeting of two relief or rarefaction waves travelling in opposite directions. Its magnitude is that of the compressive stress existing between them before their meeting.

on iron as is created by an impact at 87/50 x 10 = 15 ft/sec on lucite. And a 53 ft/sec blow on a rigid wall would be equivalent to a 61 ft/sec impact on iron.

c. Effectiveness of Buffering

A relatively thin layer of soft material placed between the impacting bone and hard object can reduce the peak stresses induced in the two colliding bodies. For such buffering to be effective, though, the interface stress must be relieved quickly--before reverberating shock waves in the soft layer going back and forth between the two hard bodies build up the stress to the level it would have reached immediately in the absence of the soft layer. A few complete reverberations in the interposed buffer may result in an unattenuated peak stress; specifically, in the absence of relief the fraction f of unattenuated stress built up in the thin, soft buffer after n full reverberations can be shown to be given by the formula:

$$f = 1 - R^n, \quad n = 1, 2, 3, \ldots \tag{18}$$

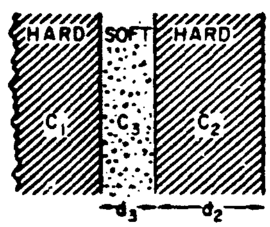
where

$$R = \frac{(I_2 - I_3)(I_1 - I_3)}{(I_1 + I_3)(I_2 + I_3)} \tag{19}$$

I_1 = impedance of one hard mass
I_2 = impedance of other hard mass
I_3 = impedance of soft buffer

provided $o < I_3 < I_1, I_2$. As n increases, $f \to 1$.

If U_i and d_i represent shock speeds and layer thicknesses, respectively, with the subscripts corresponding to the various materials as shown in the sketch, then for effective buffering



$$(n - 1)\frac{2d_3}{U_3} \geq \frac{2d_2}{U_2} \tag{20}$$

assuming that relief will come from the right and that the right-hand hard material is to be protected from an impact by the left hand material. Quantity n in this case is computed from Eq. 18, given a tolerable fraction f of unattenuated peak stress.

d. Buffering by Scalp

Goldman and von Gierke (Ref. 26) gives the acoustic impedance of "soft tissue" as 1.7×10^5 dyne sec cm^{-3} which contrasts with 6.0×10^5 for "fresh, compact bone." Furthermore, the depth of scalp over the exposed areas of the head is about the same as the underlying skull thickness. Thus if the first hard body is iron

$$R = \frac{(0.6 - 0.17)(4.0 - 0.17)}{(0.6 + 0.17)(4.0 + 0.17)} = 0.51$$

and the least possible value of f is 0.49, corresponding to n = 1. Taking shock speed equal to sound speed in both bone and scalp means that

$$U_2 = 3.4 \times 10^5 \text{ cm/sec}^* \quad \text{and} \quad U_3 = 1.5 \times 10^5 \text{ cm/sec}$$

Since $d_3 \approx d_2$, inequality (20) shows that $n \geq 1.44$ and that relief from the right will come before n = 2. Hence the scalp may reduce peak stress by half. The presence of brain tissue to the right of the second hard layer could conceivably reduce the effectiveness of the scalp by preventing complete relaxation.

The sound speed and impedance used above for "tissue" are very much like the corresponding values for water. If there are similarities between the shock properties of scalp and water, then the effectiveness of scalp buffering may be sharply reduced at high pressure because the shock impedance of water rises strongly with increasing pressure. At 50 kbar (5×10^{10} dyne/cm^2), for example, it is 4.5×10^5 dyne sec cm^{-3} instead of 1.5×10^5. While pressures as great as 50 kbar are not of interest here, it should be kept in mind that the effective shock

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* Compressional wave speed.

impedance of tissue may be in some cases significantly higher than the corresponding acoustic impedance. Using $I_3 = 3 \times 10^5$ in Eq. (19) for example changes R considerably:

$$R = \frac{(0.60 - 0.30)(4.0 - 0.30)}{(0.90)(4.3)} = 0.29$$

and

$$f = 0.71 \quad .$$

e. Countermeasures

Although the scalp may not be a highly effective buffering agent in the sense of reducing peak impact stress, Table IV suggests there may be artificial means readily available. A layer of wood, for example, placed over heavy concrete structural members in a shelter might increase the threshold for head or brain injury several times. In view of the ignorance surrounding the mechanism of head injury, any quantitative calculation of the effectiveness of buffering in reducing injury must be doubtful; in fact, until the critical time interval for brain injury can be identified, effectiveness must be determined largely empirically. Work of this kind has been done by Dye (Ref. 70) who as a result recommended a padding material for use in prize fighting rings. There is no reason such a material would not be quite practical in underground shelters. Furthermore, as will be seen later the most likely kind of injury in below ground shelters is that resulting from toppling, and against such injury ring padding should be effective. Whether installation of that particular countermeasure would heighten hazard from other less likely modes of injury or whether there are even more suitable measures remains to be studied.

Other evaluations of buffering effectiveness have been made. Campbell (Ref. 71) in a statistical study of matched pairs of similar auto accidents reports "a significant association between presence of padding (on instrument panel) and lesser head injuries." Buffering or padding was not helpful in reducing the threat to life in these auto collisions and the degree of lessening in injury hazard was not dramatic.

This does not mean that more effective padding can not be designed. Helmets are widely worn by motorcyclists and appear to be a worthwhile protective measure (Ref. 72). Again, the protection offered is not complete. However, the threats involved in road accidents are much greater than those we are considering here; against motion hazard in shelters protective measures would seem to be very effective.

f. Skull Fracture Threshold

We have already in an earlier section found skull fracturing at impact speeds of 19 ft/sec in the auto crashes studied by Swearingen (Ref. 30). The objects struck by the heads of these victims are not easily generalized. Padding appears to have been slight but in two fracture cases the object seems to have been sheet metal rolled into the roughly cylindrical form of the dashboard. Such a configuration may be capable of giving the striking body considerable resistance.

Other studies have been made in the laboratory.

Gurdjian, Webster, and Lissner (Ref. 73) dropped fifty-five intact human cadaver heads onto a heavy steel slab so that blows were struck in specified areas of the skulls. In some cases skull bending patterns were recorded with stresscoat. From the weight of each head and the height of fall required to produce fractures in each position, these investigators report thresholds for fracturing in the front midline, the back midline, top midline, and in the region above the ear in terms of the energy dissipated by the impact. The least energies in their fifty-five member sample were 415 in.-lb, which produced two midfrontal fractures in a 10-lb head, and 400 in.-lb, which fractured a 10-lb head as a result of a blow to the occipital region. Speeds of both heads at impact were 14.7 ft/sec. Among the rest of the cases 6 out of 10 mid-front fractures were produced by speeds less than 15 ft/sec and three out of nine blows to the occipital area led to fracture thresholds below 15 ft/sec. Thresholds for blows to other areas were all above 15 ft/sec. (27 cases).

This threshold, 14-15 ft/sec, is slightly more conservative than that we found above from Swearingen's accident simulation as is to

be expected from impacts on solid steel but also considerably more conservative than our rough calculation based on a reported value of the tensile strength of "bone"; all however are clearly of the same order of magnitude. Since evidence suggests that the threshold for injury to a specific part of the skull or brain in a specific individual is fairly narrowly defined (within 20%, Ref. 73), and since civil defense shelters will house people of all kinds and ages, the lowest plausible value for a threshold is the only one pertinent to the present study.

The large disparity between the calculated spall fracture speed for intact head impact on iron, i.e., 53/(0.87 x 0.74) = 82 ft/sec, and the observed speed for fracture in a head with intact scalp, i.e., 15 ft/sec, indicates fairly clearly that the simple spall mechanism is not the limiting consideration for tolerance to impact of the human head. Skull fracture may be associated with motion of a relatively large area of skull (but not necessarily the whole skull) which may be a flexure of a zone that builds up unsupportable tensions along the boundaries of the zone.

Stresscoat studies reported by Gurdjian, Webster, and Lissner (Ref. 73) suggest that fracture starts at a distance from the point of impact and reaches the impact area only after rebound of the struck bone takes place. First, cracks seem to appear on the inside skull surface and are always radial suggesting excessive hoop stresses. The bending mode responsible for failure has been sketched by these writers and their sketch is reproduced in Fig. 21. Fractures in an area of the skull diametrically opposite to the impact have been seen also (contrecoup), but the authors

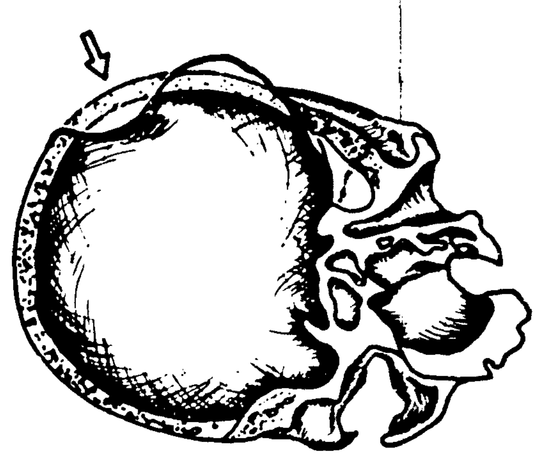


FIG. 21 BENDING OF SKULL ASSOCIATED WITH SKULL FRACTURE (from Ref. 73)

say this does not seem to be the usual response. Gurdjian and his co-workers found they were able to predict the location of fractures associated with certain impacts and thus could attach strain gages to the skull before impact and study surface motion as a function of time. They report first motion at the fracture site 0.60 msec after impact and start of fracture 1.2 msec after impact. Since transit time at 3.4×10^5 cm/sec across a 12-inch skull is much shorter than this, the most fruitful approach to understanding skull fracture mechanism does not seem to be through shock wave theory; however, cracking does begin well before one period (3.3 msec) of the fundamental flexural mode has elapsed. The. exact understanding of the skull fracture mechanism remains a subject for further study.

For now we will have to accept an empirical limit of 15 ft/sec on speed of head impact with a nearly rigid body and keep in mind that such an impact must last more than one or two msec to cause fracture. Striking sheet steel at a speed above threshold for example may not have the same effect as striking solid steel. Ordinary car glass will not sustain dangerous impact stress more than a few microseconds, and is not a source of head injury (Ref. 74).

Gurdjian, Webster, and Lissner also report that the skull without skin and flesh covering breaks with one tenth the energy of impact as the fresh cadaver head. The writers do not describe any experimental basis for this statement but if the fleshless skull were dropped from the same height onto the same surface as the head with flesh, the impact energies of the two would be in the approximate ratio of one to ten (the ratio of their weights) while the impact speeds would be equal. The lack of flesh could certainly lead to a higher stress in the fleshless skull than in the other, but the difference, as we have calculated, is not large. Since the fleshed head is heavier than the skull, the duration of the pressure on it is longer than on the bare skull; on the other hand, the scalp and viscera may offer resistance to flexure. Unless they compensate each other these two features of the fleshed head are apparently without pronounced effect.

It seems common sense to look upon a blow following a cut in the scalp as more dangerous than one delivered to an intact area of the scalp. However, scientifically, the question still appears open.

g. Blow from a Sharp Object

It should be noted that all the foregoing discussion has concerned blows from blunt objects. Dye (Ref. 75) states that whereas 600 in.-lb of impact energy will fracture a skull when the striking body is flat, only one tenth as much or 60 in.-lb is required to fracture the skull when the body has the shape of a 90^0 corner. Dye describes no experimental evidence nor the degree of injury resulting from the low energy blow. He does not state whether a break in the scalp is an essential feature of the effect. It would seem likely that a fracture from a sharp object could be confined to a very small area contrasted to that stemming from impact with a blunt missile. But if the means of fracture is a specific bending mode of the skull it is hard to see exactly in what way shape of the striking object is important. There are, of course, many kinds of fractures.

The question of the importance of cutting the scalp with the edge has already been mentioned. Aside from this effect, there is hydrodynamically a better chance of penetration or fracture by a sharp object than by a blunt object of the same mass and speed because the time average stress at the interface between object and target must be higher when the object rests against a small area of the target than when the interfacial area is large.

2. Shock Wave Injury to Other Body Parts

It is reasonable to suppose the shock wave viewpoint can be applied to injuries to other parts of the body than the skull. The skeleton and even flesh are capable of sustaining shock motion.

a. Foot and Ankle Bones

Black, Christopherson, and Zuckerman (Ref. 27) dropped cadavers feet first with locked knees from various heights onto a flat hard surface and concluded that an impact speed of 11 ft/sec or more injured the bones

of feet or legs. Durkovic and Hirsch (Ref. 28) analyzed a wartime mine attack on a small ship at sea and found that two men standing stiff-legged on deck suffered ankle and heel fractures as a result of a sudden upward thrust of the deck at 10 ft/sec (Deck material not stated). Durkovic and Hirsch estimated that the upward acceleration of the deck lasted 6.5 msec.

Using the Goldman and von Gierke value for the impedance of fresh bone, 6×10^5 dyne sec cm^{-3}, we calculate the peak stress induced by impact of bone and a rigid surface at a relative speed of 11 ft/sec as

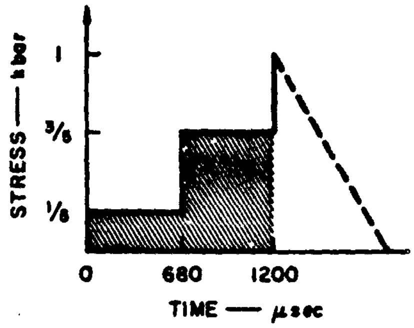
$$\sigma = 6 \times 10^5 \text{ dyne cm}^{-3} \times 11 \times 12 \times 2.54 \text{ cm}$$

$$= 2.02 \times 10^8 \text{ dyne/cm}^2$$

Since this is about one-fifth the tensile strength of bone, at least two more reflections of the impact shock wave (each carrying a speed change of 11 ft/sec) must be made before bone fracture can be expected; that is, the initial impact of heel and deck causes a shock front to move up the leg carrying a speed change of 11 ft/sec and a stress level near 1/5 kbar. At the hip the wave diverges and weakens so that whole change is not communicated to the rest of the body, which essentially impacts the top of the leg bone inducing a downward-going shock of magnitude only slightly less than the one which just reached the hip from the deck. At the deck reflection occurs and stress behind the second upward moving front is about 3 times σ or 3/5 kbar. Since shock travel time to the hip is about 300 μsec at 3 mm/μsec, a stress of 1 kbar is reached at the heel after at least 1200 μsec. Actually each reverberating shock is weaker than its predecessor so the whole process of stopping the body takes longer than 1.2 msec.

If the above simplified view of bone loading is correct, the impulse transmitted to a falling man through the heel in the three steps illustrated below should equal his initial momentum.

If the area of impact A is taken as 1 in.^{2},* then the impulse can be calculated from the area under the curve at the right.



$$A \int F\,dt = 1/5\ \frac{14.7 \times 10^3 \times 32 \text{ lb ft}}{\text{in}^2 \text{ sec}^2}\ 6. \times 10^{-3} \text{ sec } 1 \text{ in.}^2$$

$$+ (3/5)\ 14.7 \times 32 \times 6 = 2.2 \times 10^3 \text{ lb ft/sec}$$

This is close in value to the momentum of a 165-lb man striking the rigid surface at speed of 11 ft/sec, viz.,

$$mv = 165 \times 11 = 1.8 \times 10^3 \text{ lb ft/sec}$$

Actually because of the gradual degradation of the reverberating pulse more reflections and a longer time are necessary to stop the falling body than are suggested by this estimate. It appears that Swearingen's and Hirsch's volunteers (points M, N, O, P, and Q in Fig. 13) came close to suffering broken feet.

The foregoing analysis is not precise enough nor its use of physiologic knowledge great enough to establish the site of bone fracture,

---

* The contact area between one heel and the surface is estimated at 1 in^2. In the work of Black and others it is not clear that precautions were taken to insure simultaneous impact of both heels and the two heel injuries reported by Durkovic and Hirsch befell men who were both standing on one foot. A second shock wave travelling up another leg would of course reduce by a factor of 1/2 the impact of the hips upon the top of any single leg bone.

• As far as ultimate stress in the bone is concerned the presence of a thin layer of soft flesh and skin between deck and bone is inconsequential in this case.

but it does make the hazard plain and can be used in regard to other impact locations. Seated men may be exposed to a danger from the upward thrust of a hard chair. However, because of the greater area of contact between chair and body than between heel and deck, the threat of pelvic bone fracture appears at a higher relative speed than 11 ft/sec. On the other hand, the backbone would seem to be almost the sole transmitter of the shock to the chest and head, and of course the vertebrae are small in cross-section; but data from the underwater explosion near the mine layer indicate rather strongly that the tolerable speed change for seated men is above 10 ft/sec. Of 12 seated men on the ship all received blows corresponding to speed changes of 10-17 ft/sec but only one was injured; his head suffered "concussion"--probably when it struck a bulkhead. Thus his injury is very likely not related directly to the acceleration applied through the backbone. The reason for the greater shock tolerance of seated men contrasted to standing men is not clear. The articulations between the pelvic bone and the lowest vertebra and between neighboring vertebra are different than the articulations in the foot and leg; that is, a less rigid connection may transmit very much lower shock stresses into the vertebrae than are aroused in the foot of the standing man. In other words, momentum in the upper torso of the seated man may be built up slowly. If this is so, the simple oscillator model is more useful than shock wave concepts in dealing with this failure mode. In principle either model applies; certainly vertebrae are fractured when aircraft crewmen are ejected at high speed for example, but unless the breakage occurs during the first few reverberations of a loading wave along the spinal column, the phenomenon is best treated as a quasi-static loading effect and the spring model applied. The tolerance limits for this mode of damage have been covered in an earlier section from the point of view of the equivalent oscillator.

b. Flailing Arm or Leg

A flying or flailing arm or leg without the inertia of the body acting on one end can strike a flat hard surface broadside at considerable speed without bone fracture. In fact the foregoing analysis would suggest that speeds near $5 \times 11 = 55$ ft/sec are needed to produce fracture stresses

in the bone under conditions of simultaneous impact. Bruising, even destruction of flesh, may come before bone breaking. When the limb is not fully supported, however, the situation is more like that described above in connection with heel and ankle fracture. Localized stress buildup occurs due to what is essentially repeated impact caused by the ongoing unrestrained part. If the supported area A is small and the mass of ongoing limb large, the peak force reached at the support rapidly becomes large and fracture is more likely than if A is large and the unsupported mass small.

Table V summarizes what is known of the sensitivity of the body to shock wave damage in terms of the relative speed between a body structure and a rigid flat surface needed to break the bone. A value of minimum duration of the unrelieved interface stress is also suggested.

Table V

SENSITIVITY OF BODY STRUCTURES TO SHOCK WAVE DAMAGE

	Relative Speed (ft/sec)	Duration (msec)
Foot of standing man	10	≥2
Pelvis of sitting man	>17	(?)
Head	15	10-30
Arm, leg, rib broadside	>55 (?)	>0.02

Bruising of the flesh and muscle covering of the skeleton has been considered relatively unserious. Certain abdominal visceral organs, e.g., kidneys, may be damaged by local impacts which possibly might be best treated by shock wave concepts but this effect has not been studied since it seems to require rather special impact conditions.

VII COMPARISON OF HUMAN TOLERANCE LEVELS WITH LIKELY ENVIRONMENT: ASSESSMENT OF HAZARD

No human or primate has lived through a nuclear explosion while located at a point where the peak airblast pressure was much over 1 or 2 psi, and where the ground motion is strong enough to be itself hazardous. Thus, the evaluation of ground motion hazard to humans must proceed indirectly. The tools for such an approach have been described in the foregoing pages.

Hazard can be taken up under two headings: the threat to unrestrained men and that to restrained men who move essentially with the shelter. Collision and injury to free man may come through primary impact with the shelter moving under the influence of the ground motion or through secondary impact suffered after the individual has lost his normal posture and topples or is thrown against the shelter or a heavy object in the shelter. Toppling may be especially dangerous and is treated separately.

A. Pure Airslap and Unrestrained Men

Primary impact under pure airslap motion is not likely to be dangerous. The vertical thrust is down away from the man at a speed slow enough to prevent a substantial free fall into the floor. For example, if the downward ground motion is a steady 3 ft/sec,* the body will reestablish contact after

$$V_M t = \frac{1}{2} g t^2$$

$$t = \frac{2V_M}{g} = 0.19 \text{ sec}$$

Collision speed is less than $gt = 6.1$ ft/sec, which is not in itself dangerous (see Fig. 13).

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* The highest peak value likely at 50 psi; see Chapter V, Section D.

Horizontal impact between a body structure and a massive shelter component at a relative speed of 3 ft/sec is also not to be feared normally. Such a speed also seems unable to provoke whiplash injury (Fig. 13).

During a 1-mt surface explosion the positive phase of the 50-psi overpressure airblast wave lasts approximately 1 sec (Ref. 38). Thus, the free man is out of contact with the downward-going floor during only a small part of the time it is moving. With one important difference the downward motion of the free body is generally the same as that of the floor. The difference is the peak acceleration: reasonably as high as 30 g for the floor but only 1 g for the man. In Fig. 22 we have drawn

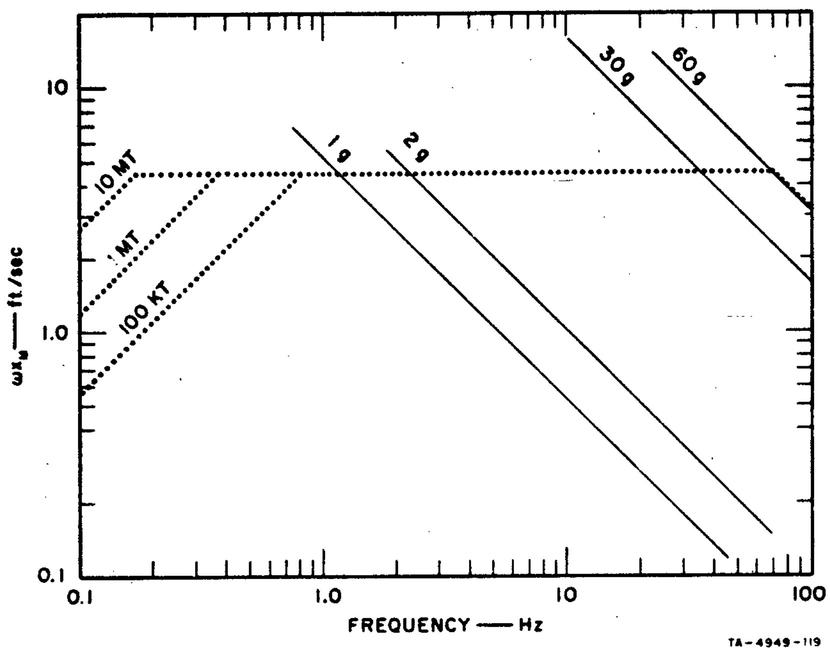

FIG. 22 MAXIMUM AIRSLAP RESPONSE SPECTRUM AT 50 psi (undamped)

an airslap response spectrum for the 50 psi range by means of the trapezoidal rule based on peak acceleration = 30 g and peak ground speed = 3.0 ft/sec. Displacements for three yields were calculated by the

simple (and conservative) rule based on overpressure duration.* Also appearing in the figure is the locus of constant acceleration equal to 2 g in order to show the large effect the value of peak acceleration has even in the frequency range 1 to 10 Hz. By lowering the peak downward acceleration from 30 to 1 g responses of all the important bodily systems are rendered harmless. For the complete response spectrum we should add the effect of the recontact pulse, but by calculating the speed of recontact we have already appraised its effect.

For example, the ground motion spectrum of Fig. 22 implies a peak spring distortion $(4.5/2\pi 20) \times 12 = 0.43$ inch (1.1 cm), which according to the results of Section VI-C might be dangerous to the head-neck system. Lowering peak acceleration to 1 g, however, reduces the distortion rate by almost a factor of 10, i.e., to 0.5 ft/sec, for which the peak strain is within even Dieckmann's bound for steady oscillatory excitation.

At the remaining body frequencies the indicated strains due to unmodified airslap are not dangerous as indicated by the comparisons in Table VI below between values of peak spring distortion from Fig. 22 and from Table III.

Table VI

MAXIMUM THREAT IN PURE AIRSLAP MOTION AT 50 PSI

Frequency (Hz)	Max. Tolerable Distortion, x_0 from Table III (inch)	Max. Distortion, from Fig. 22 (inch)
2	75	4.3
3.5	24	2.5
10	2.1	0.86

*

$$D = \int_0^{T_1} V dt \text{ or } D_m = \frac{1}{2} \frac{\Delta P}{P_o U} T_1$$

Since the 1 g downward peak acceleration of the free man has not been taken into account, Table VI also estimates the threat to restrained men at the frequencies shown. These tabulated values are independent of weapon yield above 10 kt. Below 10 kt the threat is even less.

As we have seen the resonance in the third row of Table VI is compound, including the two skeletal modes at 4-5 Hz and 10-15 Hz for standing men and at 4-5 Hz and 20-30 Hz for sitting men. The entries in row 3 are meant to describe the conservative envelope of the two modes, at least approximately. We have not been able to distinguish in the data any difference between such envelopes for the standing and sitting postures.

The bodily mode abstracted in the second row of Table VI is secondary, involving chest heaving and visceral motion. Presumably its asymptotes are included within the conservative envelope noted above. However, this is not certain; there is no unchallengeable physical evidence to locate the long duration asymptote because the chests of individuals undergoing sled and centrifuge tests are usually restrained by straps and all that can be said of the short duration asymptote is that from Hirsch's and Swearingen's data it is 11 ft/sec or more.

B. Ground-Transmitted Motion and Unrestrained Men

The peak speed carried by the ground-transmitted wave which was chosen as a basis of calculation in the foregoing chapter, i.e., 8 ft/sec, was an extreme, the most thought likely. But at such an extreme the danger of impact injury to free men in the pure rolling ground wave would be just beginning--provided we neglect the very strong likelihood that standing men will unlock their knees and sitting men will take some of the load on their arms. At the upward crest of each swell a body resting on the floor may lose contact with the floor which will be regained by impact after a short free flight. The violence of the impact will be determined by the relative speed of the body with respect to floor at moment of recontact.

If floor motion follows Sauer's idealized curves, Fig. 10 (Ref. 1), for the ground-transmitted component, there is no danger of a body moving more than a fraction of an inch out of contact with the floor provided

$$\frac{V_M}{T_2} \leq 4$$

where V_M is peak floor speed in feet per second and T_2 is the characteristic time of the motion in seconds. As V_M/T_2 increases to 7.5, recontact speed increases slowly from zero to V_M, and over the range.

$$9 \leq \frac{V_M}{T_2} \leq 14$$

recontact speed will reach its peak in the range 1.6 to 1.7 V_M, after which it decreases slowly, as indicated by the entries in Table VII. (Toppling of course is not considered.)

Therefore the rolling motion may conceivably endanger unrestrained people standing stiff-legged or seated in hard chairs, but the likelihood will depend on geologic environment and yield; namely, injury requires both $\sim 8 < V_M/T_2 < \sim 20$ and $V_M \geq \sim 6$ ft/sec.

Table VII

RELATIVE RECONTACT SPEED IN GROUND-TRANSMITTED (outrunning) MOTION

$\frac{V_M}{T_2}$ $\left(\frac{ft}{sec^2}\right)$	≤4	7.5	9.1	14	17	≥26
Relative recontact speed	0	1.0 V_M	1.6 V_M	1.7 V_M	1.5 V_M	1.0 V_M

As noted earlier, the value V_M = 8 ft/sec has been chosen as the highest likely peak speed in the rolling component by extrapolating the peak downward ground speed under airbursts in Nevada back toward ground zero to the 50-psi range, see Chapter V, Section D.

The duration of the free flight hypothesized above will be one-half or more of T_2 and in many cases should be long enough to permit relaxation of locked knees by an alert individual. A man seated in a hard chair could have more trouble protecting himself, however.

The most dangerous kind of rolling wave then (as far as recontact is concerned) is a short duration strong wave occurring near the beginning of outrunning. There is no hazard in ground-transmitted motion of a magnitude "proven" by measurements at Eniwetok or the Nevada Test Site.

On the basis of the idealized speed history for vertical motion, the threat from the upward thrust can be analyzed as that due to a one-sided acceleration pulse of peak magnitude (1/4) (V_M/T_2) (g) and duration $(1/2)T_2$. Average acceleration approximately (1/8) (V_M/T_2) (g) and speed change 2 V_M. Generally such a pulse will be harmless because of the low average acceleration; in fact only if T_2 has its least value, 0.1 sec, and V_M its highest likely, 8 ft/sec, will average acceleration approach the hazard level for seated men, i.e., 10 g. The corresponding asymptote for acceleration applied through the feet is not known but on the basis of standing man's natural frequencies appears to be in the same neighborhood as that of the seated man.

C. Restrained Men in Pure Airslap Motion

As pointed out above, the only possible threat to restrained men arising from purely airslap motion falls on the head-neck system possessing a natural frequency 20-30 Hz in vertical oscillation and a half natural period equal to 15-25 msec. Pulling the shoulders of a restrained man down away from the head in the airslap motion may produce neck pain and constitute a marginal hazard in extreme geologic environments since the impulsive asymptote for this injury mode (6 ft/sec, Fig. 13) is near the highest likely ordinate shown in Fig. 22 (4.5 ft/sec) at 20 Hz.

Since peak horizontal airslap motion is even less than vertical, the whiplash limit (V_o = 6 ft/sec in Fig. 13) will not be exceeded.

Because it is a very brief transient, the 30 g downward acceleration in the airslap pulse poses no threat. The relatively low values of A_o in Table II are long duration asymptotes.

D. Restrained Men in Ground-Transmitted Motion

Over a range of characteristic times T_2 between 2/2.5f and 10/2.5f, Fig. 9 shows a possible spectral ordinate amounting to four times peak ground speed. Thus if V_M = 8 ft/sec, the ordinate may reach 32 ft/sec at frequencies threatening to the body provided

$$0.04 < T_2 < 1.3 \text{ sec}$$

If the ground-transmitted or seismic wave does appear ahead of the airslap at the 50-psi radius such bounds on T_2 include all possible values of T_2 at the 50-psi range for yields of 1 mt or less. A spectral ordinate of 32 ft/sec is eight times greater than the highest possible due to airslap as shown in Fig. 20.

Table VIII

POSSIBLE AND TOLERABLE STRAINS

Frequency (Hz)	Max. Tolerable Equivalent Distortion, x_o, from Table II (inch)	Max. Equivalent Distortion from Fig. 9 (inch) V_M Peak Ground Speed (ft/sec)		
		8 ft/sec	6 ft/sec	2 ft/sec
2	75	34 in.	21 in.	7.6 in.
3.5	24	20 in.	12 in.	4.4 in.
10	2.1	6.9 in.	4.3 in.	1.5 in.

Multiplying the entries in column 3, Table VI, by 8 brings both those in the second and third rows near or above their opposite entries in the second column and thus introduces the possibility of hazard to the corresponding bodily subsystems (see Table VIII). The threat to the bodily subsystem represented by the third row (10 Hz) appears most severe

from this calculation; Fig. 9 indicates that one of the arbitrary wave combinations can be found to which a 10 Hz system will respond with a response spectral ordinate 4 times maximum base speed if duration of the whole motion comes within the range of 0.2 to 1 sec. That the duration will fall within such a range is a strong possibility.

Similar deductions from Fig. 9 can be made regarding the sensitivity of the head-neck subsystem of a torso-restrained individual possessing a frequency 20-30 Hz. Here even the "proven" level of motion in the rolling component may threaten since $V_o = 6$ ft/sec; the peak spectral ordinate for $V_M = 2$ ft/sec may be 8 ft/sec and thus hazardous to the head-neck subsystem. But to offer such a threat the total duration of the wave must be unusually short, viz., at least less than 0.4 sec. At the speculative level, $V_M = 8$ ft/sec, the pain or injury threatened is more severe but the wave duration must again be less than about 0.4 sec.

To summarize, the skeletal frames of restrained individuals will be threatened by the presence of ground-transmitted motion of the "extrapolated" or "speculative" amplitude and of a certain range of durations. The likelihood these durations will occur is high. The neck on the other hand is marginally threatened by a rolling component of "proven" magnitude but the chance of the occurrence of the necessary threatening durations is low.

E. Modification of Threat in Ground-Transmitter Motion with Peak Overpressure and Yield

If observations are made at a range corresponding to a given peak overpressure and if outrunning begins at a lesser range determined by a certain peak overpressure, then in most cases the characteristic time T_2 of the rolling motion is proportional to the cube root of yield and the observed intensity will presumably be independent of yield. (A departure from proportionality may enter because of the constant term in the expression for the characteristic time T_2, Eq. 1, Chapter V, Section D). The effect of making observations at a range where peak overpressure is lower is to reduce the intensity of motion and to increase its characteristic time. The peak spectral ordinate would be lowered

and moved toward lower frequencies compared to its original size and position. The amount of reduction of the ordinate is directly proportional to reduction in peak ground speed, which by Eq. 4, Chapter V, Section D, falls inversely as the square of the scaled radius. For example, the peak ordinate at 23 psi is one-half whatever it is at 50 psi. At 100 psi, if the ground-transmitted motion is still outrunning the airblast, the peak ordinate is about double. If outrunning begins between 100 and 50 psi, the maximum spectral ordinate at 100 psi is due solely to airslap and can conceivably be less than that at 50 psi, although such a condition has not been seen either in Nevada or Eniwetok.

When outrunning depends on relatively deep earth strata, the relation between yield and characteristic time T_2 becomes more complicated. Sauer (Ref. 1) gives a nomograph for locating the point of outrunning in terms of peak overpressure. It is a fair approximation to assume the overpressure at outrunning increases in proportion to the increase in the cube root of yield. For example, if for 1-kt outrunning occurs at 100 psi, then for a yield of 1-mt outrunning takes place at a range slightly greater than the 1000 psi radius. Thus even if low-lying earth layers are responsible for outrunning, the tendency for T_2 to increase with yield remains, but the rate of increase is a little less than otherwise.

F. Contribution of the Shelter Response to Hazard

In Chapter V, Section E, we found that the spectrum of the motion to which shelter contents are exposed can in theory be significantly intensified by the response of elements of the shelter structure, although no such intensification has yet been demonstrated during weapons tests. The frequencies intensified according to the theory are the natural frequencies of the elements. The amount of the magnification of the spectral ordinate appears to be something between a factor of 2.5 and 10.

1. Restrained Individuals

Structural frequencies 50 Hz and above are probably of little effect on human hazard at 50 psi for two reasons: the dangerous rolling wave can have only a weak component at this high frequency to be magnified

and the hump raised on the otherwise harmless airslap spectrum at 50 Hz does not influence the spectrum dangerously in the range 2 to 25 Hz where human sensitivities are located.

On the other hand, structural resonances in the range 5 to 20 Hz could in certain geologic environments be enormously dangerous since the structures concerned might easily vibrate in sympathy with strong ground-transmitted waves.

For instance, a rolling wave of "proven" amplitude, 2 ft/sec, normally has a maximum spectral ordinate $3 \times 2 = 6$ ft/sec at a frequency $f = 4/2.5T_2 = 1.6/T_2$. Magnification of this peak then occurs on any structural element of natural frequency f_n where

$$\frac{1}{2} \times \frac{1.6}{T_2} < f_n < 2 \times \frac{1.6}{T_2}$$

For T_2 equal to its least value, 0.1 sec, these bounds become

$$8 \text{ Hz} < f_n < 32 \text{ Hz}$$

Many of the structural elements postulated by Agbabian-Jacobsen (Ref. 33) fall in this range. Complete evaluation of the threat in this case depends critically on the amplification factor chosen; but even a factor of 2.5 will, under certain conditions, lead to a marginal threat to the restrained body since $2.5 \times 6 = 15$ ft/sec, which is greater than the longitudinal skeletal asymptote $V_o = 11$ ft/sec.

A rolling wave of "extrapolated" or "speculative" amplitude, 8 ft/sec, which is dangerous to restrained individuals without amplification probably becomes deadly under an amplification of only 2.5, i.e.,

$$8 \times 3 \times 2.5 = 60 \text{ ft/sec}$$

On structural elements of frequencies in the range from 5 to 20 Hz the maximum airslap spectral ordinate, 4.5 ft/sec, would be only marginally threatening under amplification of 2.5 but could be extremely dangerous if the amplification is as large as 10.

2. Unrestrained People

If airslap excited vibrations in a floor at a frequency of 25 Hz or less and the floor continued to vibrate for a minute or more with a small amplitude after the shelter inhabitant came again in contact with it (after the downward thrust) then the person could find the motion "intolerable" in the sense used by Dieckmann (Ref. 10). Such shaking need not be of large amplitude but cannot be strongly damped. For example, according to Dieckmann, steady vertical vibration at 10 Hz with maximum floor or seat displacement of 2 mm is bearable for at most 1 min. Vibrations at an amplitude of 10 mm are "unbearable." Dieckmann does not describe injury, if any, resulting from the exposure.

A structural member can, of course, be put into vibration by a ground-transmitted wave as well as by airslap and any shaking persisting while the inhabitant is in contact with the member could be "intolerable."

If we assume a likely but minimal value for damping, say 5% critical, duration of structural vibration can be calculated under the most pessimistic ground motion conditions. Suppose the free field airslap spectral ordinate is 4.5 ft/sec at all pertinent frequencies and the free field ground-transmitted spectral ordinate is 32 ft/sec at all pertinent frequencies, which are the frequencies dangerous to humans, i.e., 5, 10 and 25 Hz. Maximum spring distortion, i.e., relative displacement of the floor from its normal orientation, is found as

$$x_o = \frac{(\omega x_m)}{2\pi f_n}$$

where (ωx_m) is the spectral ordinate, chosen pessimistically above. Time for a free vibration initially of the amplitude given by x_o to be damped out to a harmless level, say 1 mm, can then be calculated from

$$x = x_o e^{-\nu 2\pi f_n t} = 1 \text{ mm}$$

(p 200, Ref. 5)

Results are shown in Table IX below.

Table IX

TIMES REQUIRED TO DAMP OUT THREATENING SYMPATHETIC VIBRATION, $\nu = 1/20$

f_n (Hz)	Maximum Airslap (ωx_M = 4.5 ft/sec)			Roll: extrapolated (ωx_M = 32 ft/sec)			Roll: proven (ωx_M = 8 ft/sec)		
	x_o (ft)	x_o (mm)	t(sec)	x_o (ft)	x_o (mm)	t(sec)	x_o (ft)	x_o (mm)	t(sec)
5	0.14	43	2.4	1.0	300	3.6	0.25	77	2.8
10	0.071	22	0.98	0.50	150	1.6	0.12	37	1.15
25	0.028	8.5	0.27	0.20	75	0.55	0.048	15	0.34

The table shows there is no chance of dangerous "steady" vibrations occurring after the exciting ground motion has passed.

During the excitation, on the other hand, there is apparently a chance an unrestrained body which has lost contact with the floor may collide with the vibrating floor at a relative speed increased by the vibrations. The amount of the increase will be of the same order of magnitude as the spectral ordinate, which pessimistically we have set at 32 ft/sec. Even the "proven" ordinate, 8 ft/sec, can be marginally threatening. In other words, any sympathetic floor vibration of an amplitude between the two values of x_o given in Table IX for the rolling motion provides platform speeds which could injure the human body. Whether or not under any given conditions an inhabitant could come into contact with the platform when it is moving with a dangerous speed requires further analysis.

In summary, there are three ways in which structural shaking or shuddering could harm an unharnessed sheltered population: by steady, small amplitude motion, by enhancing the recontact speed of body and floor during a few very strong oscillations, and lastly by behavior between these two extremes, i.e., an oscillation lasting a few seconds of an amplitude ten times greater than those explored in vibration tolerance studies yet not great enough to be treated as single pulses.

The first has been ruled out by consideration of damping; the second is a distinct possibility in environments where ground-transmitted motion is above the proven magnitude; and there are no data from which to judge whether the third has any physiological importance.

It should perhaps be repeated that characteristic vibration or shuddering in underground structures exposed to nuclear blast have not been seen. The question of structural response to nuclear blast-induced ground motion is still under active investigation. It is clearly one aspect of this analysis which tends to reduce the degree of conservatism which can be attributed to it.

G. Shock Wave Hazard

As has been indicated before, by shock wave hazard we mean the injury potential of those mechanisms which can most easily be described in terms of shock wave concepts. Shock wave injury may be conveniently discussed in terms of four modes: direct transmission, bodily reverberation, free fall, and toppling. Free fall has been covered in an earlier section of this chapter but the other modes will be discussed below.

1. Direct Transmission

The wave of motion moving through the ground can itself be considered a shock wave. When the motion is ground transmitted (as opposed to that due to airslap) the risetime in the shock "front" is unusually long--sometimes greater than characteristic body periods. Idealization of such a wave as a sudden shock will be conservative; that is, it will tend to overestimate the shock damage to the body. Although airslap shock waves rise in times more often than not of the same or smaller magnitude than bodily periods, the only significant motion they induce is downward; for unrestrained occupants the only pertinent parameter connected with airslap shock is then total displacement. Although airslap shock decays relatively fast, conservative policy will be to regard it as of constant strength during bodily response of a restrained occupant.

A man standing on an underground floor, leaning against a wall or sitting in a chair may feel a direct shock. Aside from bruises and

superficial wounds the shock may pass into the skeleton or skull and, if the induced pressure can rise into the neighborhood of 1 kbar, there will be danger of bone fracture. However, if the peak ground surface pressure is only

$$50 \text{ psi} = 0.003 \text{ kbar}$$

there is no danger of bone injury in the direct shock arising from air-slap on the nearby surface.

There is no shock wave in the rolling component pictured in Fig. 10 and used hitherto as the basis of discussion of a "ground-transmitted" wave. The origin of this wave has been attributed to airblast remote from the point of observation. However it is conceivable ground-transmitted motion includes a shock wave. In the absence of focusing or other concentrating effects an upper bound on the strength of such a hypothetical shock wave can be given.

When 1 kt is exploded on the surface, peak airblast pressure is 50 psi at a distance of 500 ft from the explosion (Ref. 39). At a distance of 500 ft from a 1-kt explosion completely contained within a homogeneous ground mass, the peak pressure should be larger than expected near the surface of the same soil at the 50-psi range from the surface burst. Furthermore, a detonation on the surface of a homogeneous mass produces a peak pressure within the soil directly below the shot that again is an upper bound on the peak pressure at the same range along the surface. There have been two nuclear shots at the Nevada Test Site in two widely different materials providing data of this kind: one, a contained shot in tuff, yielded particle speed data which can be interpolated to 3.1 ft/sec^{-1} (0.0715 kbar) at a scaled range of 500 ft/kt$^{1/3}$ (Ref. 76); at another, a contained burst in granite, the peak pressure at 500 ft/kt$^{1/3}$ was 0.5 kbar (Ref. 79).

In addition, a third explosion Flat Top I, was carried out with 20 tons of chemical explosive on a Nevada Test Site limestone surface and, although there was considerable scatter in the measurements, it produced motion of roughly 1.5 ft/sec at a distance of 136 ft directly below the explosive center (Ref. 76). (The distance 136 ft is equivalent

to 500 ft from a 1-kt explosion.) The impedance of the limestone at this stress level is probably about 1.6×10^6 gm cm^{-2} sec^{-1}.

In estimating the stress directly passed to an occupant by shock waves in soil, conservative policy is to double the particle speed behind the oncoming wave and consider the body struck by a thick, rigid body moving at that speed. The incoming particle speed may be found from the soil impedence I and pressure P from the formula

$$u = \frac{P}{I}$$

For granite $I \cong 1.6 \times 10^6$ gm cm^{-2} sec^{-1} and for tuff $I \cong 7.6 \times 10^5$ (Ref. 77 and 79). Thus at the 50 psi range in granite

$$u < \frac{0.50 \times 10^9}{1.6 \times 10^6} = 310 \text{ cm sec}^{-1}$$

$$2u < 620 \text{ cm sec}^{-1} = 21 \text{ ft/sec}$$

and in tuff

$$2u < 6.2 \text{ ft sec}^{-1}$$

For limestone at Flat Top I the corresponding speed is

$$2\,u \cong 3.0 \text{ ft/sec*} = 90 \text{ cm/sec}$$

* The order of magnitude agreement between this speed and that used in earlier sections for the speed in the ground-transmitted wave, i.e., 2 to 8 ft/sec, is interesting but not clearly significant. The physical models are different. The rolling component of ground motion discussed earlier presumably stems from energy collected by and transmitted through a deep layer of finite thickness to a point below the target on the surface from where the collected energy is radiated into the target. Given a certain highly fortuitous but unfavorable geological environment the amount of this energy could be a much greater fraction of the total explosive energy even than that reaching the same radius outside a wholly contained shot.

The limestone and granite with very similar impedances would be expected to transmit pressure in about the same way. The difference of a factor of 7 above reflects mostly the lack of containment of the limestone burst. If the impedance of bone is taken as 6×10^5 gm cm^{-2} sec^{-1}, then the impact at 620 cm sec^{-1} stemming from the contained explosion in granite induces a stress σ

$$\sigma = 620 \text{ cm/sec} \times 6 \times 10^5 \text{ gm cm}^{-2} \text{ sec}^{-1}$$

$$= 0.37 \text{ kbar} < 1 \text{ kbar}$$

Thus a surface burst in hard rock cannot produce dangerous direct shocks.

The threat from direct shock does appear to increase with hardness of the soil material but fortunately granite and limestone are two of the materials of highest shock impedance commonly found.* However, both tuff and granite absorb kinetic energy from a passing motion wave, the tuff much more so than the granite. This has the effect of spreading the momentum in the wave over a larger volume than it would occupy without energy losses and reducing the peak stress reached within the wave. We would expect a large formation of hard, highly competent rock to sustain higher peak pressures. While the presence of such a dangerous formation near a shelter site may not be likely, the possibility must be kept in mind.

We conclude therefore that direct transmission of shocks from a surface burst into the body does not appear to threaten a shelter occupant at the 50 psi range. (This conclusion can be extended at least to 500 psi.) Such a conclusion overlooks all likelihood of focusing or reverberation of shocks within the ground. Shock waves can condense and mutually strengthen one another under certain very favorable circumstances, but the occurrence of this phenomenon on a large scale in the field has not been explored.

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* Some extremely rich iron ores and certain single crystalline earth materials can have higher impedances.

2. Bodily Reverberation

There is however a strong likelihood of reverberation in the body when the shock thrust is from below--as in a ground-transmitted wave.* When a bone supports a substantial weight attached to it--as the leg bones support the bodily weight of a standing man--the shock will reverberate in the bone and increase the stress there until the supported weight is brought up to ground speed, provided the pressure in the ground is kept up. Unless a man is carrying a heavy load the extreme case will be that of the standing man supported by one leg. This case is conservatively treated as an impact between a thick rigid body and man at a relative speed equal to twice the peak particle speed behind the incoming shock front. In other words, when the explosion is on the surface collision speed is not greater than 3 ft/sec. Validity of the analogy requires that shock pressure be maintained during the whole reverberation period of several milliseconds; field experience shows this is reasonable at this stress level in granite.

The work of Swearingen and others (Ref. 20), Hirsch (Ref. 19) and Gurdjian, Lessner and Webster (Ref. 31) sets the beginning of dangerous impact speeds near 10 ft/sec which refers to a collision of an unstressed human with an unstressed rigid surface. (There was a shock wave in the minelayer studied by Durkovic and Hirsch (Ref. 28) but the value of relative speed they reported for injurious impact was equivalent to a "free surface speed" and thus may be compared to the observations of the other writers just mentioned for injury threshold in free fall.) Except under the most bizarre circumstances the limit found above for granite $2u < 3.0$ ft/sec indicates a large margin of safety because of the neglect of relief from the free surface in computing the stress at the 50 psi range. Direct transmission of shocks into the body does not seem to threaten shelter occupants even considering the possibility of reverberation within the body.

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* Nothing like a shock wave coming from below has been seen at nuclear weapons tests. This is an extreme assumed for the purposes of argument.

H. Toppling

In the immediately foregoing section we assumed the man kept the same bodily orientation during shelter motion. Such an assumption in many cases unduly limits the likelihood of injury. Any standing man, for example, who is toppled onto a hard surface runs the risk of injury whatever the degree of ground motion.

A head falling freely a distance of $s = 5$ ft or more may strike a hard or sharp object at a speed of

$$\sqrt{2\ gs} = \sqrt{2 \times 32 \times 5} = 18 \text{ ft/sec}$$

which is above the lowest threshold for brain or head injury, viz., 10 ft/sec.

We can easily calculate the striking speed of a rigid body of length ℓ much greater than its thickness, toppling freely from an upright position onto a flat surface. We will assume (conservatively) the body does not slide against friction but rotates about its point of contact with the floor.

Since the kinetic energy during impact equals the potential energy of the original configuration

$$\int_0^{\ell} \frac{1}{2}\, \rho(\dot{\theta}x)^2\, dx = \frac{1}{2}\rho\ell g$$

where ρ is the body's linear density and $\dot{\theta}$ is the angular speed of impact. The final linear speed at the top of the toppling body is then

$$\dot{\theta}\ell = (3g\ell)^{1/2}$$

which for a six-foot man becomes

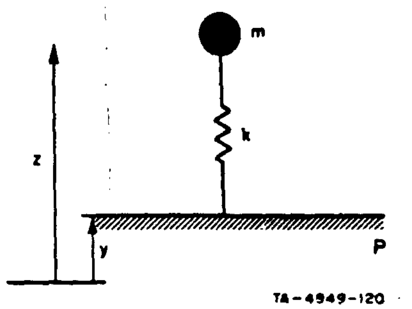
$$\dot{\theta}\ell = (3 \times 32 \times 6)^{1/2} = 24 \text{ ft/sec} \quad .$$

This is even further above the danger level for head blows than the impact speed of a freely falling head but probably below the threshold

for arm or rib fracture when the whole bone impacts the surface simultaneously and when the bone is not wholly supporting the rest of the body. Very likely this means that a person could fall with his body held stiffly and still escape serious injury by protecting his head with his arms and looking out for his elbows. Likelihood of harm could be reduced still further, however, by falling loosely so that the impact was absorbed slowly by less sensitive body parts than the head or the forearm bone. Possibility of sprains and dislocations certainly exists. In other words, although time and means are available for defensive actions, avoidance of injury may in general call for considerable skill.

Since the motion of a shelter will in general include both strong horizontal and vertical components and since the users of a shelter will be of all ages and degrees of physical alertness, it must be assumed that some people in the shelter will lose their balance during a nuclear attack.

Crede (Ref. 32) postulated conservatively that a person in a shelter would lose his balance if he became separated from the floor for any length of time. Were a man a rigid body, the downward acceleration of the floor might then reach 1 g before endangering him. Crede, however, idealized man as a mass supported by a spring attached to the floor (see sketch); since under normal gravitation the spring is compressed, downward motion of the floor at some constant acceleration less than 1 g would cause the spring to expand to its uncompressed length and any constant acceleration greater than that would lead to stretching, which Crede interpreted as loss of contact, loss of balance, and injury. The critical value of constant acceleration turns out to be 1/2 g, as can easily be shown as follows. Man's body is represented by the mass "m" and the floor by the platform, P; spring constant is k.



Downward displacement of mass m in a fixed coordinate system is z; and displacement of P is y. Thus if "a" is the constant downward acceleration of P,

$$\ddot{y} = a, \qquad a < g$$

The amount of stretching of the spring beyond its unstrained length ℓ_o is

$$\ell_o - (y-z)$$

which if gravity provides the total force acting on m:

$$m\ddot{z} = mg - k[\ell_o - (y - z)]$$

If

$$x = y - z$$

then

$$z = a - \ddot{x}$$

and

$$\ddot{x} = a - g + \omega^2(\ell_o - x)$$

$$\ddot{x} + \omega^2 x = a - g + \omega^2 \ell_o$$

where $\omega^2 = k/m$.

Writing the solution as a sum of particular solution of the original equation plus a general solution of the homogeneous, we find

$$x = A \cos \omega t + B \sin \omega t + \frac{a - g}{\omega^2} + \ell_o$$

Since at $t = 0$:

$$x = \ell_o - \frac{mg}{k} = \ell_o - \frac{g}{\omega^2}$$

and $\ddot{x} = 0$, then

$$x = \frac{a}{\omega^2} \cos \omega t - \frac{g - a}{\omega^2} + \ell_o \quad .$$

Maximum stretching of the spring takes place when $\cos \omega t = -1$ or

$$x_{max} = \ell_o - \frac{g - 2a}{\omega^2}$$

Thus if $a > 1/2\ g$, the spring is stretched beyond its original length.

As noted earlier, peak acceleration in pure ground-slap motion is almost two orders of magnitude greater than this at the 50 psi range and remains greater until peak overpressure has fallen to a harmless level. Even the rolling ground-transmitted motion has been seen to carry a peak acceleration of 3 g at the 50-psi range or 500 $ft/kt^{1/3}$ (Eq. 2, Chapter V, Section D).

Crede's criterion appears to be extremely conservative on two accounts: because he supposes toppling at downward accelerations less than 1 g and because he assumes that whoever is toppled is injured. However, when strong horizontal motion is simultaneous with a downward thrust, loss of footing becomes much more likely than if there is only vertical motion. Crede very likely felt that without the full weight of the body pressing on the floor, the frictional forces tending to prevent horizontal slippage were weakened and when this frictional bond was suddenly reestablished a strong horizontal impulse tending to topple could be imparted to the man. Certainly, keeping one's balance under

the ground motion conditions contemplated may well be a matter of considerable skill not available to every shelter occupant. Furthermore, falling, when unavoidable, so that injury is lessened or avoided may also call upon abilities not present in all shelter occupants.

VIII EQUIPMENT SENSITIVITY

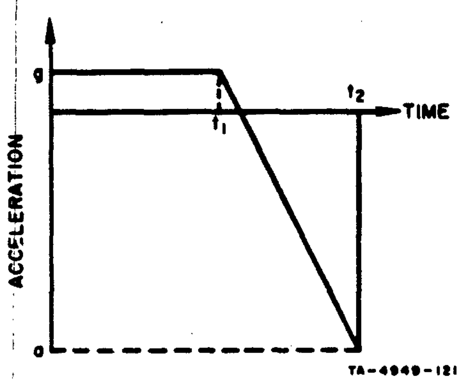
Variation between humans is slight compared to the differences between, for example, a diesel generator and a teletype machine. Moreover there are different sizes and manufacturers of each kind of equipment. General remarks then about equipment sensitivity to strong ground motion are hard to make. It can be stated though that most of the shock and vibration environments in which shelter equipment (e.g., radio receivers and transmitters, telephone switchboards, fans, valves, pumps, electric motors, batteries) can be found normally differ strongly from the underground motion environment during nuclear attack. Normally equipment will suffer small displacements--fractions of inches--and in drops and falls may undergo relatively high peak accelerations of hundreds of g's. Underground motion is long-lasting and results in large displacements.

Knowledge of equipment pertinent to an assessment of the ground motion threat is not generally available. Usually, hardened sites have been designed and outfitted under a philosophy demanding isolation, and equipment for the site tested only for its performance with a particular isolation system. Thus tests for shock and vibration tolerance have generally been long-lasting, low-amplitude, low-frequency periods of oscillation. A typical such test may use the free oscillation of a damped spring which vibrates at the resonant frequency of the isolation system. A more severe test is performed on a shake table vibrated sinusoidally for 5 minutes during which time the frequency is swept smoothly from 5 Hz to 55 Hz and back to 5 Hz. Peak acceleration is limited to 3 g or less. This test may incidentally provide simulation of the damaging characteristics of the undulatory or ground-transmitted portion of some nuclear-induced ground motion at some sites. It is, of course, easy to imagine sites and attacks where the soil response would be different--of greater amplitude or longer period than 1/5 sec or both. And for all site locations the test undoubtedly is overly severe, i.e., it

subjects the test object to motions not likely to arise in a nuclear attack at any one site. (This test however has not so far as is known been used for forecasting the reaction of hard mounted equipment; it is applied to equipment which will be isolated by an isolation system of unknown frequency.) Moreover, none of the usual shake table tests will adequately reproduce the airslap motion.

A. A Good Simulation of the Motion Environment

Drop tests can be devised which will show response spectra enveloping typical spectra from the airslap component of nuclear explosions and in fact matching certain airslap envelopes very closely. These consist essentially of allowing a heavy platform carrying the test object to fall on a conical lead pellet or dropping a heavy platform with blocks underneath into a box of sand; the form of the acceleration history at the platform to which the test object is tightly fastened is sketched in the figure. If "A_M" is the peak deceleration, t_1 the time the test object strikes the cone, and t_2 the time the object comes to rest, then, since the final speed is zero,



$$A_M = \frac{t_1 + t_2}{t_2 - t_1}$$

where

$$t_1 = (2s/g)^{1/2}$$
$$s = \text{drop height.}$$

For a given platform and test object the height of drop and the cone angle determine the peak deceleration, A_M, reached. A computed, undamped response spectrum* envelope corresponding

* Computation is made by assuming the base of a single degree-of-freedom undamped linear oscillator moves with the acceleration history shown in the inset. For each oscillator frequency ω peak mass speed both during and after the transient motion is derived. The peak speed during the strictly harmonic vibration following the transient motion is the quantity ωx_M where x_M is the peak displacement in the harmonic area.

to a drop of 16 ft* and deceleration duration of 0.1 sec is reproduced as curve A in Fig. 23; the dashed line in the same figure (curve B) is an envelope consisting of the three straight lines:

$$\omega x_M = 1.5 \text{ times maximum speed of base, i.e.,}$$

$$1.5\left(\frac{t_1 + t_2}{2}\right)^2 \frac{g}{t_2} = 48 \text{ ft/sec}$$

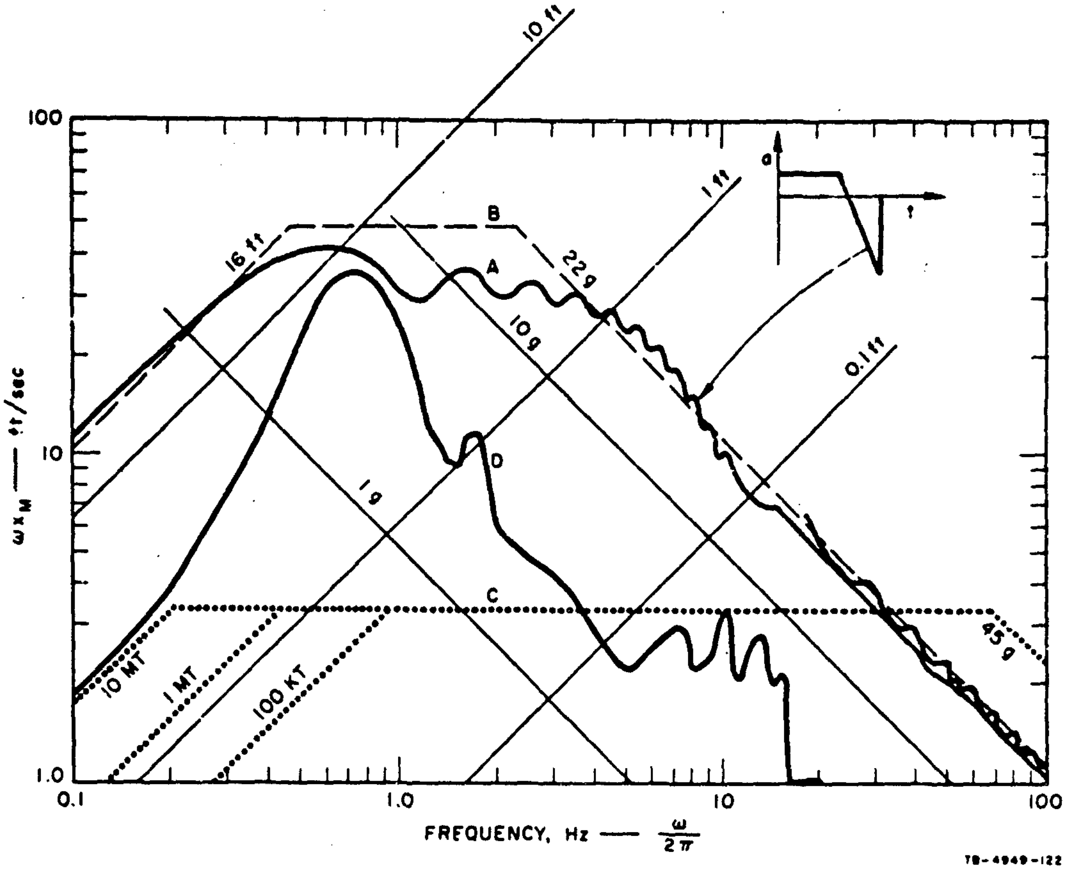


FIG. 23 DROP TEST RESPONSE SPECTRUM COMPARED WITH NUCLEAR SPECTRA (undamped)

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* The actual drop height can be considerably less than 16 ft if artificial acceleration is supplied (by springs, for example) during the early phase of the drop. This lowers the spectrum at low frequencies but the 16 ft free drop specified is more than adequate here.

x_M = s = maximum displacement of base, i.e., 16 ft

$\omega^2 x_M$ = twice maximum acceleration A_M, i.e., 22 g .

Like airslap ground motion spectra, spectra from these drop tests can be approximately described by three straight lines in coordinates of ($\log \omega x_M$, $\log \omega$). To insure survival during airslap, a test object must be subjected to an impact the spectra of which envelop all likely airslap spectra. (This is the fundamental assumption of most shock testing. For a brief discussion of why it may not be adequate, see Paszyc, Ref. 36.) For example, from Sauer's compilation (Ref. 1) of the peak airslap effects at Frenchman Flat at the 50 psi range, namely, vertical speed, 2.5 ft/sec, and vertical acceleration, 22.5 g, the three envelope boundaries have been drawn in Fig. 23 as curve C. (Each of the low frequency bounds corresponds to a different yield, 100 kt, 1 mt, 10 mt.) The envelope of the test motion does not include the actual ground motion envelope and a better test for the postulated airslap would involve a higher acceleration, a, and a lower maximum speed. This could be achieved by a shorter drop and a wider angle cone. But drop height may not be lowered beyond the maximum soil displacement expected in the field. A 10-mt surface burst, for instance, on Frenchman Flat soil may move the surface downward by airslap at the 50 psi range by an amount not greater than 1/2 uT where u equals 2.5 ft/sec and T, the positive phase duration, is about 2.8 sec; that is, the least drop height, if 10 mt were the largest weapon expected, is 3.5 ft. There does not seem to be any difficulty in principle about devising a drop test whose spectra would envelop the airslap spectra from such a burst.

Curve D in Fig. 23 is the spectrum computed from a hypothetical ground-transmitted wave produced by a 10 mt surface explosion; T_2 = 1.735 sec and V_M = 8 ft/sec. From the figure the drop test appears to be capable of providing a simulation of the rolling motion as well as airslap although the particular test specified is overly severe; for the shelter environment and attack corresponding to the spectrum shown, the stopping cone may have a narrower apex than the one specified. Other stopping devices

than a lead cone are available and may produce more suitable simulation. Vigness gives a very brief description of some of these devices (Ref. 6). It seems clear that drop tests can be devised to envelop the spectrum for any likely ground motion at the 50-psi range. Tests must be carried out with the object mounted both vertically and horizontally.

(Since there are no likely equipment sensitivities below 2 Hz, spectrum D presents no additional threat over the airslap spectrum C. However, were T_2 to have its least possible value, i.e., 0.100 sec, and V_M still 8 ft/sec, the peak of curve D would fall near 1.5/0.1 = 15 Hz and curve D would exceed curve C for all frequencies below approximately 37 Hz, which clearly includes sensitive frequencies of some large equipments. The 16 ft drop test would be inadequate to simulate such an extreme ground-transmitted wave but could be made adequate without increase of drop height by the use of springs to help propel the test object into the arrestor.)

Ideally a single test should embrace the whole nuclear response spectrum, but if a shelter resonance (most likely found in the range 5 to 50 Hz) with low damping is strongly excited by the ground motion, the kind of test suggested in Fig. 23 may not be adequate. Separate tests on shake tables and spring tests may have to be used to cover a narrow frequency range defined by a resonance. Such a procedure of sequential as opposed to simultaneous testing in different frequency ranges is not adequate if there is strong coupling between modes at frequencies sequentially tested.

B. Existing Test Procedures

Unfortunately, reports of the results or even ready availability of drop tests of the magnitude described above have not been found in the open literature. Reported test procedures fall short in two ways: height of drop is less than that required and capability for handling the heaviest and bulkiest equipment found in shelters is lacking. However, the airslap pulse can easily be simulated in all but the largest pieces of equipment by means of drop testers described in military specifications. Items weighing up to 1200 lbs and measuring up to 5 ft in cross-section

can be tested in free falls up to 6-1/2 ft by the "variable duration" shock tester (Ref. 78). Such a machine would not be applicable to the largest diesel generators which serve as power sources in emergency operating centers nor to the large pieces of refrigerating equipment which might be used. Components could easily be handled of course but separate testing of components is adequate only as long as the coupling between them is not influential. Blowers, ordinary water and sewage pumps, switching equipment, batteries, and other pieces are also suitable for this standard test.

The tests most commonly reported, however, seem to be those of the "impact" family, which are also delineated in military specifications (Ref. 79). Testing machines come in two sizes for testing: light weight and medium weight equipment; there is a third procedure involving underwater explosions for heavy equipment. Maximum equipment cross-section allowed on the larger of the two machines is again 5 X 5 ft but highest weight accepted is 7400 lb. Highest equipment displacements attainable during testing are 3 inches (Ref. 32), an order of magnitude below that expected in some nuclear blast environments. Peak accelerations obtainable are said to be near 2000 g. Maximum spectral ordinates match those occurring in the spectrum of the strongest likely ground-transmitted motion (Fig. 23) but do not appear to occur in the same part of the frequency spectrum, falling instead at frequencies in the range from 50 to 100 Hz (Ref. 32). The heavy equipment test involving underwater explosions is a much more satisfactory test of behavior under exposure to nuclear explosions. Data given by Oleson (Ref. 80) imply a trapezoidal response spectrum corresponding to a maximum displacement of 1 to 2 ft and spectral ordinate 9 ft/sec or more. Again, there are shock-wave or very high frequency components but they appear to be considerably degraded or filtered by the floating platform to which the equipment under test is mounted, and Oleson reports the high frequency spectral bound to lie typically at $\omega^2 x_M = 150$ g.

The three testing methods described in Ref. 79 stem from the U.S. Navy's interest in the chance of damage aboard ship from underwater explosions. The two machines involve hammer blows upon a platform or anvil

to which the test object is tightly attached. Much of the damage from these hammer blows should be akin to what we have discussed earlier as shock wave effects, and test results may constitute an upper bound on what shock wave injury might be expected in a shelter. For small pieces of equipment (relatively unaffected by low frequency motion) this failure mode may be the most important. Since hammer falls are never more than about 5 ft in these machines, the highest shock stresses in the test subject will likely be in the range 0.1 to 1 kbar (1470 to 14,700 psi). Converted to cgs units, 50 psi becomes only 0.003 kbar, several orders of magnitude smaller than the test stresses. Hence, survival on the test machine means good likelihood of survival in the field--as far as shock damage due to airslap is concerned. Measurements of peak acceleration in the range 200-1000 g are reported for these hammer devices but such values must be an average during a time interval a tenth or more of a millisecond long. Thus when the test object is rigidly attached to the struck plate, the reported peak accelerations may actually be exceeded for very short periods.

Peak displacements reached with the light weight high impact (LWHI) hammer machine are near 1.5 inches; thus equipment frequencies below about 5 Hz may not be stimulated to maximum speeds as high as $1.5 \times 2.6 = 3.9$ ft/sec (which Nevada Test Site data suggest may be necessary at some locations), but such frequencies are likely only in a small number of equipments, e.g., lightweight bulky pieces. For larger equipment the medium weight high impact (MWHI) hammer machine provides maximum displacements of 3 inches, thus adequately simulating airslap for equipment with frequencies as low as 25 Hz but not of course the strongest likely ground-transmitted components. Response spectra for the hammer machines given by Dick (Ref. 81) and by Dick and Black (Ref. 82) show that peak speeds and maximum displacements in spectra of both machines decrease rapidly as the height of the hammer drop is reduced below 1 ft. Depending on the weight of the attached equipment the peak speed varies generally between 50 ± 15 in/sec and 100 ± 20 in/sec for hammer drops between 1 ft and 5 ft, the highest allowed. For the same lengths of hammer drops maximum accelerations on the struck platform or

anvil are, of course, very much higher than anything likely to be found in a nuclear environment and such high frequency components in the simulations probably only obscure the reaction of the tested equipment to the nuclear induced motion.

The variable duration medium impact (VDMI, Ref. 78) drop tester appears to be a better test for simulation of airslap motion than the hammer blows. Very high frequency components are more easily controlled and adequate peak speeds and displacements are easily reached. When the falling platform is stopped by the penetration of hardwood blocks into loose sand, partial response spectra have been computed by Crede (Ref. 32). The trapezoidal rule applies very well for these spectra.

Whether or not the equipment has sensitivities needing testing at the low frequencies and large displacements where existing test methods appear to fail is moot. So far as is known, no one has tried to calculate resonant periods of pertinent equipment theoretically or made a systematic experimental study to uncover them. Without stating the source of their information Agbabian-Jacobsen Associates (Ref. 33) list certain equipment and their natural frequencies in horizontal motion (Table X). The most threatening region of the response spectrum for ground-transmitted motion

Table X

NATURAL FREQUENCIES OF SOME SHELTER EQUIPMENT*

Equipment	Weight (lb)	Horizontal Natural Frequency (Hz)
Microwave R and T	130	50
Base station transceiver	300	25
Tape unit	800	10
Air compressor	300	40
Generator set	1170	20

* After Agbabian-Jacobsen Associates, Ref. 33.

shown in non-dimensional units in Fig. 24 can at most embrace only the two lowest of these natural frequencies since according to Sauer (Ref. 1) $T_2 \geq 0.1$ sec; that is, since the abscissa at the peak ordinate in Fig. 24 is 3, for the peak spectral ordinate to threaten the tape unit

$$T_2 = \frac{3}{2.5 \times 10} = 0.12 \text{ sec} \quad ,$$

and to threaten the generator set

$$T_2 = 0.06 \text{ sec} \quad .$$

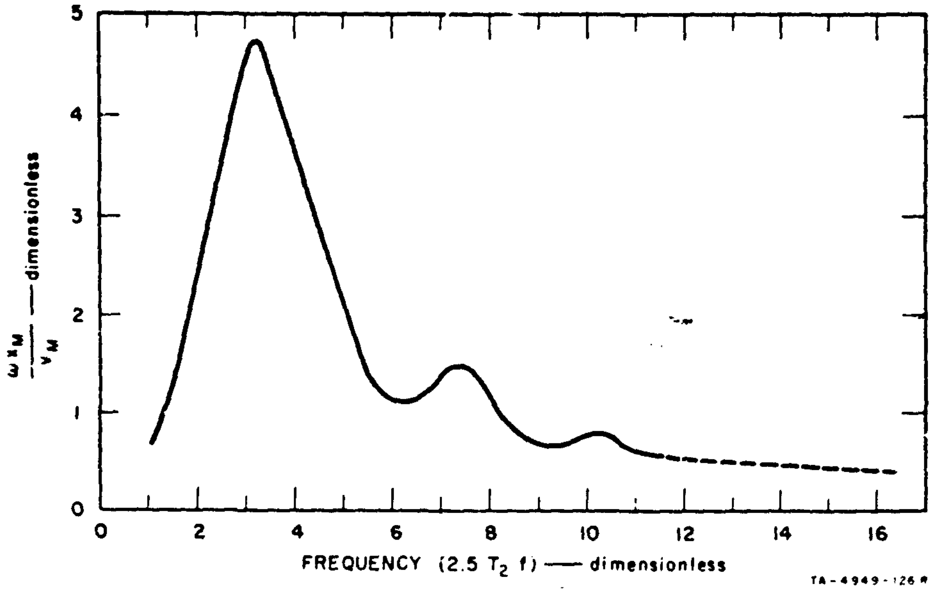


FIG. 24 RESPONSE SPECTRUM FROM GROUND-TRANSMITTED MOTION

Such a threat to the other items in Table X require $T_2 \ll 0.1$ sec. Unlike the human body much inanimate equipment has immunity from the most severe effects of the rolling motion. However, Table X contains no large cabinetry, such as found in air-conditioning equipment or in racked communications apparatus, no large motors with their mounting platforms.

Small objects, e.g., radio receivers, emergency battery powered lights, and switches, would seem to be relatively immune to the components of motion at frequencies below 10 Hz and tests of such items can very likely be accepted as adequately severe. In regard to larger accessories of shelters and command centers the question appears open at present. The Naval Civil Engineering Laboratory (NCEL), Port Hueneme, California, is considering the design of a large motion generator to be used in the stimulation of nuclear blast ground environments (Refs. 33 and 36). Motivation for creation of the device stems not only from the need for a means of testing large equipment to large displacements, but also from the doubts of Paszyc (Ref. 36) concerning the reliability of the spectral envelope as an indication of equipment response; Paszyc points to the possible need for reproducing waveshapes when the reacting load is a complex network of harmonic and non-harmonic systems. In his view, engineers have not yet demonstrated the adequacy of the spectrum envelope technique of testing when applied to specific items of apparatus subjected to complex motion. Horizontal and vertical motions should also be imparted simultaneously. In their simulations NCEL hopes to take into account responses in the shelter structure itself. No existing testing method or machine comes close to meeting all these requirements.

Bell Telephone Laboratories has recently written test specifications* as part of a general program looking toward hard mounting of communications equipment in special installations such as hardened long-lines repeater stations, but apparently isolation is still the general practice of the Bell system.† The specifications consider three components of the nuclear induced ground motion: airslap, the roll transmitted through the ground and the "shudder" or response of the structure to the airblast. The aim is to produce in separate tests motions whose cumulative effect will be a spectrum enveloping the three motion spectra as they are

* R. W. Crawford, Bell Telephone Laboratories, Whippany, N. J., personal communication.

† C. L. Wickstrom, Pacific Telephone and Telegraph Company, San Francisco, California, private communication.

presently known. Original theoretical and experimental research to define the shuddering response is underway.* An interesting and important feature of the specifications is a search through frequencies 1-10 Hz for resonant frequencies in the equipment that might overlap frequencies of the building in which the equipment will serve and, if any are found, a severe shake table test of the equipment in the neighborhood of the overlapping resonant peaks.

C. Existing Test Results and Other Data

Much shock testing has been done by the U.S. military and the Ralph M. Parsons Co. has a good compilation of many of these results (Ref. 32). The range of equipment and the goals of testing have not always been strictly pertinent to our purposes but a brief summary of this work will be included here. C. D. Morrissey reports tests made specifically on equipment destined for installation in the New York State Emergency Operating Center (EOC) at Albany (Ref. 83). Most of these tests were made on the standard military test gear. One exception was a simple impact of two railway flatcars, one of which carried the diesel generator for the EOC. Morrissey's collection of test documentation will be summarized here as well† (Table E-1, Appendix E).

The Bell System does not appear to have tested generally for hard mounting. Their tests have in the past been designed to reveal the performance of the equipment together with its isolation system. If the equipment fails a given test, the isolation system is usually changed. As pointed out above, this attitude in the Bell System may be changing and exploration of the chances of hard mounting is being undertaken. In their hardened sites, the Bell System presently puts electronic and switching equipment in spring mounted bays or on pendulum suspension from the ceiling (with snubbers to limit the swing). Large wet-cell

* J. W. Foss, Bell Telephone Laboratories, Whippany, N. J., personal communication.

† C. D. Morrissey, Praeger-Kavanagh-Waterbury, Consulting Engineers, New York, N. Y., private communication.

batteries and motor-generators are spring mounted. Their goal has been equipment survival in nuclear attack environments at high (unspecified) overpressure but they do not claim the equipment will operate during the attack itself.*

In addition to their use of testing machines, the U.S. Navy has made a study of a wartime mine-attack on a mine-sweeper (Ref. 28). Some data on equipment response was collected and will be noted below.

1. Military Tests with Shock Test Machines (Ref. 32)

Electronic equipment appears to be remarkably sturdy. Oscilloscopes pass hammer tests which are undoubtedly more severe in regard to their high frequency or "shock wave" character than airslap motion in a buried location. In one test of a frequency standard a crystal is broken--probably as a result of this high frequency component. The sensitivity of such equipment to the low frequency content below 20 Hz suggested by the ground motion spectra of Figs. 9 or 24 is probably low, but special concern should be shown for the racks in which the equipment may be mounted.† Such a method of installation may introduce resonances in the range 0.5-10 Hz where the ground motion spectrum is strong.

Other equipment tested in this same way and brief summaries of the results are listed as follows: batteries in hard rubber jars survived, one battery in plywood lost its seal; three out of five circuit breakers, 50 to 2000 amperes, failed by changing from open to closed or vice versa; a fuse box was unhurt as was a gear motor for helicopter lift; various relay and control panels passed, but two out of three separate relays failed, one by changing position, the other through damaged electrical contacts; wire wound and various subminiature resistors survived; two out of three limit switches passed; two out of four rotary switches failed when components were distorted; roughly one-third of miscellaneous

* C. L. Wickstrom, Pacific Telephone and Telegraph Co., San Francisco, California, private communication.

† R. W. Crawford, Bell Telephone Laboratories, Whippany, N. J., private communication.

switches failed mostly by breaking or bending of components; two large refrigeration fans, 8 inch diameter, with 115 volt electric motors failed (in one the blade locked against the guard, in the other a base plate buckled); a 75-lb heat exchanger was unhurt as were various kinds of valves ranging from 3 to 80 lb in weight.

Failure of switches by changing position is not important unless the change has more serious secondary consequences in the circuit of which it is a part. At any rate, it is a sign of the possible operation of frequency components other than the very highest, as indeed are the bending and distortion noted above.

During other tests with the larger of the two test machines, equipment was isolated to some degree from the direct blow* and the injuries may not have been of the shock wave type. Reported peak accelerations at the mountings were, however, still in the range 100-500 g. The reactions of heavy equipment tested in this way are mixed: a 300 ft^3 compressor and its gasoline engine, a diesel engine frame, nine out of ten centrifugal pumps with their electric motors and one 100 HP electric motor all passed Navy impact tests of this kind. The failing motor and pump was 5 HP, 400 gallons/min., and sustained electrical damage. Other pumps--both larger and smaller--seem to have been unharmed. (Most of the equipment was required to keep running during and after the test.)

In all the foregoing testing with the Navy hammer impact machines, spectral envelopes were far outside any likely ground motion envelope at frequencies above 40 Hz and, although unreported below 40 Hz, were probably well within the highest likely ground motion envelope below about 10 Hz. In the tests highest spectral speeds and accelerations were between two and three times the required levels. Whether more failures in the equipment would have been stimulated had the machines provoked greater displacements hence lower frequencies than they did is not

* A certain degree of isolation arises because the equipment is attached to steel beams which themselves are attached only at their ends to the struck member. [These beams also tend to amplify frequencies in the range 50-60 Hz (Ref. 84)].

known. Whether the failures that were observed were the results of the excessive severity of the tests above 40 Hz is not known.

2. The War-Time Attack

Section VIII-C-2 is contained in Appendix F and is classified Confidential. This material is at the time of publication of this report undergoing classification review with the object of declassification.

3. Emergency Operating Centers in Albany and Oklahoma City

Suppliers of equipment to the emergency operating center which is designed to function as the alternate seat of state government in Albany were required to certify their installations for shock resistance or provide suitable isolation (Ref. 83). The standard of performance furnished by the owner was a trapezoidal response spectrum which, perhaps in part because of the hardness of the soil surrounding the site, was somewhat less demanding than that we have characterized as the maximum likely airslap spectrum for the 50 psi range (Fig. 12). Peak vertical speeds and displacements in this spectrum were just under 20 in./sec and 5 in., respectively. Maximum vertical and horizontal acceleration were taken as 18.8 g. Although peak horizontal speeds and displacements were somewhat less than the values for the corresponding horizontal parameters, equipment was required to meet the same standard in all axes. If the principle of response spectra envelopment is accepted, such a standard of performance can very likely be established by use of the standard military testing machines (Refs. 78 and 79), and the two hammer-blow machines (LWHI for lightweight and MWHI for medium weight equipment) were used for the bulk of the testing. The Air Force drop test (VDMI) supplied some of the data.

For items not suitable for attachment to the standard testing machines shock tests were improvised. A heavy, bulky motor-generator was put on a railroad flatcar and rolled into stationary cars to provide a simulation of horizontal shock motion; the two ends were then dropped individually onto wooden blocks to give vertical shocks.

In some cases shock testing was done to help the design of isolation, that is, to provide a known safe level of impact below the expected

maximum. Some suppliers could not certify their equipment above the level needed for ordinary long distance shipment (which is usually taken to mean a peak acceleration of 3 g) or some other previously established peak acceleration.

C. D. Morrissey of Praeger-Kavanagh-Waterbury, architects and engineers, New York City, has made documented shock resistance data collected during the construction of the Albany EOC available for this study, and a summary can be found as Table E-1, Appendix E, along with a brief list of equipment (Table E-2) whose manufacturers certified tolerance levels below the expected maximum. (How these levels were found is not always clear.) Comments in Table E-1 stem from consideration of the possible installation of the equipment without shock mounts in the most severe nuclear environment likely at 50 psi, as has been developed elsewhere in this report, and do not reflect any judgment of the adequacy of the testing for the Albany EOC. The diesel motor-generator for example was certified by the documented testing for a shock tolerance below the expected maximum and was therefore installed in Albany with isolation. Various approved laboratories carried out the work set forth in Table E-1.

Somewhat similar requirements appear to have been laid down for the city and state EOC's in Oklahoma City and some of the resulting documentation has been provided by Paul Sprehe, consulting engineer, Oklahoma City, and appears in Table E-3, Appendix E.

Two noteworthy points emerging from the test documentation connected with the three EOC's are: the ruggedness and suitability for hard mounting of much equipment ranging in size from lighting fixtures to 100 HP motor-pump sets; and the possibility of modifying equipment as deficiencies are found. Although the diesel generator was fairly seriously impaired during testing, the source of the failure under such loading was easily located and removed. The motor controller cabinet showed a resonance at 9 Hz but the structure was simply modified so that the resonant frequency became 25 Hz (which frequency was more amenable to testing). Various mounts which broke under test were strengthened.

In contrast to the military tests of a fan reported above, all the fans in Tables E-1 and E-3 passed the tests applied, one of which was a severe hammer blow.

As noted in the tables there are two or three items for which the lack of strong low frequency components in the test motion to simulate the strongest likely ground-transmitted motion might easily be important: the two waterchilling systems, a 10-ft-high electric distribution panel, electric switch gear assembly, and a 9-ft-high motor control center. There may be other items sensitive in this way, such as the 7 × 7-ft air filter, and possibly the large motor-pump sets, the two components of which are connected by a long relatively thin shaft.

Other possible shortcomings of the reported testing might be mentioned again at this time: the response spectrum concept applied to systems with an infinite number of degrees of freedom not all of which are linear is at best a rather poorly established empirical rule; and secondly, the means of attachment of the equipment affects its dynamic reaction to any load and this cannot be the same in a test as in practice.

Although most of the tests reported in Tables E-1 and E-3 give the equipment tested passing marks, it cannot be concluded that the items concerned will resist even the airslap component of motion at 50 psi in all ground environments. The tests made upon the motor generator for example were generally good but did not go to high enough levels. In some cases too great reliance seems to have been placed on measurements of peak accelerations in determining the adequacy of the test; it does not seem likely, for example, that large electrical distribution panels can be tested for airslap resistance with hammer drops less than 1 ft. Peak acceleration may have been high enough at several points on the equipment during the test performed but the low frequencies which might be present in such large objects have perhaps not been excited strongly enough.

The major difference between most of the tests reported in connection with the EOC's and the underwater mine attack lies in the duration of the pressure and consequent extension of the test spectrum to low

frequencies. Excitation of relatively low frequency components could easily have been responsible for the damage reported in items 9, 10, and 11 of Table X. These were items without counterparts in the testing reported for the EOC's. (Piping incidentally is an important part of shelter equipment and often it is not embedded but spans open space between walls where it is threatened by low frequency excitation unless specially mounted.)

Item 6, Table X, the diesel generator, has a close counterpart in the shelter testing and despite the greater load undoubtedly suffered in the mine attack this equipment showed generally similar kinds of damage under both exposures, i.e., that which can be repaired fairly easily.

We have suggested above that the failure of item 7, fire and bilge pumps, may have been associated with their peculiar sensitivity. Ordinary ship's pumps as well as shelter pumps were not reported hurt.

As for the remainder of the entries in Table X and in Tables E-1 and E-3 it is not clear whether differences in reported responses are attributable to differences in equipment or in exposure. It is not clear whether or not failures reported are due to high frequency components in the test motion that are not found in nuclear-induced ground motion. In particular items 4 and 5 of Table X, evaporator and air compressors, are seemingly similar to small electric equipments reported unhurt in Tables E-1 and E-3, e.g., transformer, electric motors, compressor, fans; but their failure under mine attack must unfortunately be interpreted as casting doubt on the adequacy of the standard military tests (LWHI, MWHI, and VDMI) until the contrary is proven. The mine explosion--with its high spectral ordinate and high peak acceleration--was more severe than airslap at any likely shelter site (except when strong building and equipment resonances coincide) but the testing reported in Tables E-1 and E-3 was not severe enough to simulate motion in the most sensitive environments.

4. Possibility of Quantitative Understanding

The example of the fan which failed under the hammer blow military test described above provides a tantalizing bit of evidence as to how test results might be understood more exactly.

The fan fails when its blade strikes the guard cage. A fundamental flexure mode of vibration at, say 10 Hz,* and a damaging relative displacement of 1.0 inch implies a speed change asymptote

$$V_o = 2\pi f_n x_o = 62.8 \times \frac{1}{12} = 4 \text{ ft/sec}$$

and an acceleration asymptote

$$A_o = \pi f_n V_o = 4 \text{ g} \quad .$$

---

* A cantilevered beam, cross-sectional depth h, width b, length ℓ, modulus of flexural rigidity EI, material density γ shows a fundamental flexural mode at a frequency (Hz) given by Ref. 5, p. 496:

$$f_n = \frac{(1.875)^2}{2\pi\ell^2}\left[\frac{EIg}{bh\gamma}\right]^{1/2} \quad .$$

Taking $E = 30 \times 10^6$ lb/in.^{2} and $I = bh^2/12$ we compute

$$f_n = (1.875)^2 \frac{h}{2\pi\ell^2}\left[\frac{Eg}{12\gamma}\right]^{1/2}$$

or

$$f_n = 3.25 \times 10^4 \frac{h}{\ell^2} \sec^{-1} \quad (h, \ell \text{ in inches}) \quad .$$

For $f_n < 10$, $\ell^2/h > 325 \times 10^3$ or if h = 1/16 in. then $\ell > 14$ in., which are reasonable dimensions for a fan blade.

On the hammer test device to a first approximation the amount of speed change is half the impact speed of the hammer; thus, a free fall of 3 ft produces a change of 7 ft/sec which is greater than V_o and average acceleration during one cycle of oscillation is greater than 4 g at the equipment mounting; hence tolerance limits were exceeded in the blow. Since failure by excessive blade flexure was indeed seen during such a hammer test when the blow was delivered in a direction to excite the flexural mode, perhaps a detailed analysis of equipment sensitivity can be made and compared with the likely shock environment. (For example, the hypothetical fan, mounted rigidly in a shelter subject to strong earth motion associated with response spectrum sketched as curve C in Fig. 23, would presumably just survive since the peak relative displacement of the equivalent single degree-of-freedom system at 10 Hz is 0.64 inch. The safety margin is not great, however. It should be noted, too, that the response at 10 Hz is the same for all realistic weapon yields unless building resonances exist in the frequency neighborhood.) A priori identification of possible important resonant modes will not be as easy in general as with the fan.

5. Earthquakes

Although most notable earthquakes provide surface motions in the epicentral region approaching in magnitude and character those which might be expected to arise from some nuclear burst, generally speaking, surface explosions of 1 mt or greater will provoke ground-transmitted undulations and even airslap motions at the 50 psi range of greater amplitude and acceleration than even quite destructive earthquakes. The U.S. Coast and Geodetic Survey strong motion seismometers have recorded many destructive quakes, e.g. (Long Beach, California, 1933; Helena, Montana, 1935; and Imperial Valley, California, 1940) and found peak horizontal and vertical accelerations at the surface near 0.3 g (Ref. 85). At Imperial Valley the total horizontal elastic displacement at the seismometer was in the neighborhood of 15 inches, and it was reached over an interval of 3-1/2 seconds. Vertical displacement was 4 inches, accomplished in 2-1/2 seconds. From Sauer's empirical correlations (Ref. 1) a 10-mt surface burst at Eniwetok Proving Ground might be expected to

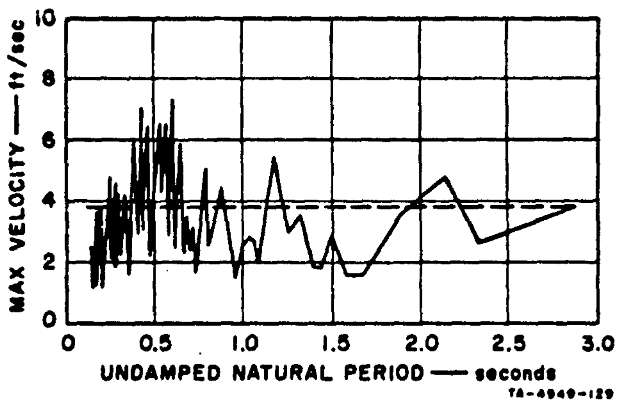
lead to total displacements at the 50 psi range of the order of 12 inches, but the rise-time would be expected to correspond to peak accelerations of about 5 g (both horizontal and vertical).

Airslap motions at overpressures in Nevada would carry roughly the same displacement but much higher speeds and accelerations. Housner (Ref. 86) and Merritt and Newmark (Ref. 2) have supplied response spectra from the seismometer readings summarized above and their work appears in Fig. 25. The dashed line in Fig. 25(a) represents generally the level of peak ground speeds expected under the airslap at the 50 psi range in Nevada. Since also the earthquake horizontal displacement was similar to the nuclear induced vertical displacement in Fig. 25(b) the usual airslap trapezoid has been entered by a dashed line. Clearly, earthquake motion may provide information on equipment response at frequencies below 10 Hz which complements that found in hammer testing to simulate airslap effects.

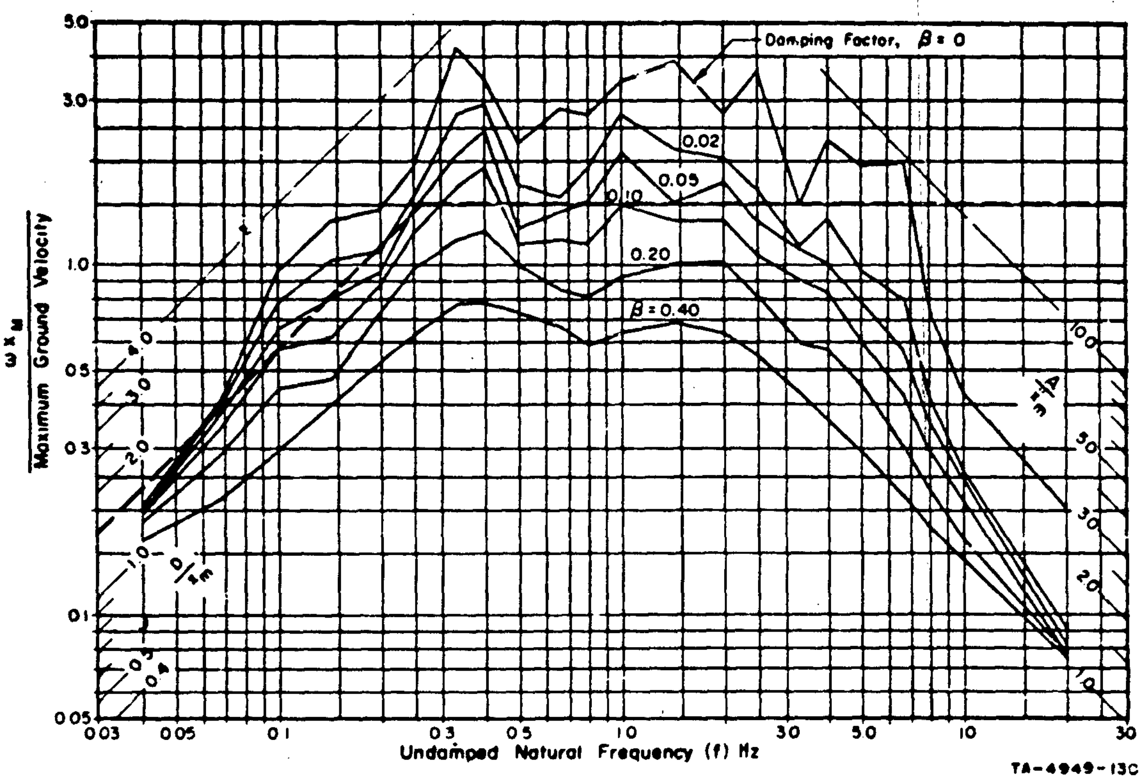
In much stronger quakes than that in El Centro in 1940, there is some chance of approaching the kind of motion characterized here as ground-transmitted. Byerly (Ref. 85) does quote observers of the Assam quake of 1897, described as perhaps the most severe on record, as reporting 1 ft high elastic ground oscillations and the upward flight of heavy boulders reaching heights of 8 ft. Although there were apparently no instrumental observations in the epicentral region, such a description implies there was at least some component of earthquake motion comparable to the nuclear induced surface motion seen at the 50 psi range in Eniwetok. Any study of effects of earthquakes on equipment would be faced with the problems of separating the influence of the shaking itself from that of fire and collision.

It may be noted in passing that the Bell Telephone system has not yet been put out of service even temporarily by California earthquakes.* Their relay racks in areas subject to earthquakes have additional bracing

* C. Shafer, Bell Telephone Laboratories, Whippany, N. J., private communication.



(a)



(b)

FIG. 25 EARTHQUAKE SPECTRA, EL CENTRO, CALIFORNIA, MAY 18, 1940 (from Ref. 86)

over that normally used, but otherwise no special isolation or mounting is required for earthquake motion protection. Generally speaking, their repeater stations contain much of the apparatus expected in a civil defense shelter except those associated with living accommodations; that is, equipment vital to their function to be found in these stations, besides telephones and telephone switchboards will be emergency battery power supplies, electronic amplifiers, electrical switches and cables, ventilating and heating fans, regular and emergency lighting systems. Heavy gasoline or oil motors and electric pumps are not usually an essential feature of such telephone centrals.

D. Conclusions

Adequate testing of equipment destined for possible hard mounting in underground emergency operating shelters is just beginning. The precise nature of the motion hazard to many important specific items is simply not known at present.

Although there are simple tests readily available which can greatly increase our precise knowledge of equipment sensitivity, these have not been widely or systematically applied to the pertinent articles. However, it seems very likely that further testing will reveal that most equipment can be hard-mounted in an underground environment at the 50-psi range from a surface nuclcar explosion, provided relatively minor structural adjustments are made. The threat to power generators and exposed piping, for example, has been demonstrated but this hazard undoubtedly can be removed by small modifications of the items themselves.

Whenever possible, alteration of equipment to remove susceptibility to low frequency excitation will greatly simplify the testing required, but some items can not so easily be changed and will have to be tested with strong low frequency motion (<20 Hz) to remove doubt as to their hardihood. Drop tests involving longer falls and longer stopping times than are widely employed now would be suitable tests of this kind. Sequential testing on shake tables of different frequencies may not be adequate although they are better than no low frequency tests at all.

Routine tests are probably adequate in general if the geologic environment receiving the equipment is demonstrably unable to support strong ground-transmitted motion. Because they produce more realistic peak stresses, routine drop tests (e.g., VDMI) are preferable to routine hammer blows (e.g., LWHI, MWHI).

Finally, the definitive answer in all cases will demand difficult and complex testing such as that proposed by the Naval Civil Engineering Laboratory, Port Hueneme, California.

Appendix A

SENSITIVITY DIAGRAM OF LINEAR OSCILLATOR

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Two cases of motion in a simple linear oscillator are treated here: (1) the forcing function $F(t)$ applied to the mass m and (2) no force on m but motion of base prescribed. The system considered is illustrated in Fig. A-1. Let y be the displacement of the mass from a fixed reference point. If the base is unmoving and F is applied to mass m directly then the equation of motion becomes

$$m\ddot{y} = -k(y - z - \ell_o) + F(t) \qquad \text{(A-1)}$$

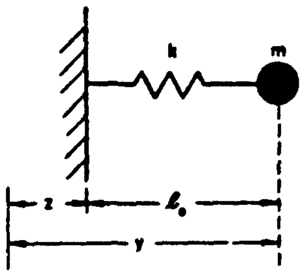

FIG. A-1

where k = spring constant, ℓ_o = equilibrium length of spring and z = the displacement of the base, is constant. Now if the displacement z from a fixed reference is given as a function time t and there is no force on m, the equation of motion is

$$m\ddot{y} = -k(y - z - \ell_o)$$

which can be converted through the substitution:

$$x = y - z$$

into

$$m(\ddot{x} + \ddot{z}) = -k(x - \ell_o)$$

$$m\ddot{x} = -k(x - \ell_o) - m\ddot{z} \quad . \qquad \text{(A-2)}$$

If in Eq. (A-1) the constant $z - \ell_o$ is set equal to ℓ_o', the two Eqs. (A-1) and (A-2) become mathematically equivalent by the substitutions: $\ell_o \rightarrow \ell_o'$, $x \rightarrow y$, and $-m\ddot{z} \rightarrow F(t)$.

The solution of the above differential equation is the sum of the particular solution which reflects the part of the response peculiar to F or $\ddot{z}$ and the general solution or solutions to the equation

$$m\ddot{y} + k(y - \ell_o') = 0 \quad .$$

General solution x_1 can be written

$$y_1 - \ell_o' = A \cos \omega t + B \sin \omega t \quad ,$$

where $\omega^2 = k/m$ and A and B are constants to be determined by starting condition on y and $\dot{y}$. The particular solution y_2 is the convolution of that part of the general solution which vanishes at time zero with the forcing function F(t) or $-m\ddot{z}$ so that the complete solution is

$$y - \ell_o' = y_1 - \ell_o + y_2 - \ell_o \qquad \text{or}$$

$$y - \ell_o' = A \cos \omega t + b \sin \omega t + \frac{1}{m\omega} \int_o^t F(\tau) \sin \omega(t - \tau) d\tau \quad . \tag{A-3}$$

Starting conditions important in this study are either: $y - \ell_o' = y_o - \ell_o'$, a small displacement due to gravity, or $y - \ell_o' = 0$ and speed $\dot{y} = 0$. Thus,

$$A = y_o - \ell_o' \qquad \text{or} \qquad A = 0 \quad ,$$

and since

$$\frac{d}{dt} \int_o^t F(\tau) \sin \omega(t - \tau) d\tau = \omega \int_o^t F(\tau) \cos \omega(t - \tau) d\tau = 0 \text{ when } t = 0 \quad ,$$

$$B = 0$$

Hence

$$y - \ell_o' = (y_o - \ell_o') \cos \omega t + \frac{1}{m\omega}\int_o^t F(\tau) \sin \omega(t - \tau)d\tau \quad . \quad \text{(A-4)}$$

If $F(t)$ is of very brief duration, viz., becomes a delta function of t such that

$$\int_{-\infty}^{\infty} F(t)dt = I_o$$

$$y_2 - \ell_o' = \frac{I_o}{m\omega} \sin \omega t$$

and the whole solution can be written

$$y - \ell_o' = A \cos \omega t + \left(B + \frac{I_o}{m\omega}\right)\sin \omega t \quad .$$

If

$$A = y_o - \ell_o', \quad \text{then}$$

$$y - \ell_o' = (y_o - \ell_o') \cos \omega t + \frac{I_o}{m\omega} \sin \omega t$$

$$= \left[(y_o - \ell_o')^2 + \left(\frac{I_o}{m\omega}\right)^2\right]^{1/2} \cos \left(\omega t + \tan^{-1} \frac{\frac{I_o}{m\omega}}{y_o - \ell_o'}\right) \quad .$$

A similar thing emerges when $F(t) = F_o H(t)$ where $H(t)$ is a unit Heaviside step starting at $t = 0$.

$$y^2 - \ell_o' = \frac{F_o}{m\omega}\int_o^t \sin \omega(t - \tau)d\tau = \frac{-F_o}{m\omega}\int_o^{\tau=t} \sin \omega\,(t - \tau)d\omega(t - \tau)$$

$$= + \frac{F_o}{m\omega^2} \cos \omega(t - \tau)\Big|_o^t = \frac{F_o(1 - \cos \omega t)}{m\omega^2} \quad .$$

Hence,

$$y - \ell_o' = \left(A - \frac{F_o}{m\omega^2}\right) \cos \omega t + B \sin \omega t + \frac{F_o}{m\omega^2}$$

or

$$y - \ell_o' = \left(y_o - \ell_o' - \frac{F_o}{m\omega^2}\right) \cos \omega t + \frac{F_o}{m\omega^2} \quad .$$

Presumably $y_o - \ell_o' < F_o/m\omega^2$ so that

$$\max |y - \ell_o'| = y_o + \ell_o' + \frac{2F_o}{m\omega^2} \quad .$$

When $|y_o - \ell_o'|$ is small, then, the very long lasting pulse of magnitude F_o and very short pulse of impulse I_o have equivalent effects on spring strain provided

$$\frac{2F_o}{\omega} = I_o \quad . \qquad \text{(A-5)}$$

The possibility of achieving equivalent effects in these two ways forms the basis of the asymptotic tolerance limits.

In case the base of the oscillator is accelerated, the quantity I_o is defined as

$$\int_{-\infty}^{\infty} -m\ddot{z}dt = I_o, \text{ that is,}$$

a delta function of acceleration produces the speed change:

$$\Delta\dot{z} = -\frac{I_o}{m} = v_o$$

and the quality F_o becomes

$$-m\ddot{z} = F_o$$

or the step in acceleration $a_o = -F_o/m$. In this case Eq. (A-5) is written:

$$\frac{2a_o}{\omega} = v_o$$

and the natural frequency f becomes

$$f = \frac{a_o}{\pi v_o} \quad .$$

When the results of experiments to find acceleration tolerances and limits are plotted in coordinates of logarithm of average acceleration (a) during the pulse and logarithm of speed change (v) effected by the pulse, the loci of constant pulse duration are straight lines of unit slope and durations (T) correspondingly to each point in the (a, v) space are given by the ratio

$$T = \frac{v}{a} \quad .$$

Hence, if the tolerance limit of a harmonic oscillator to a very short impulsive load on the base is established as a maximum allowable speed change V_o and the tolerance to a long lasting base acceleration is A_o then the lines $a = A_o$ and $v = V_o$ in (a, v) space meet at the point (A_o, V_o) where the duration

$$T = T_o = \frac{V_o}{A_o} = \frac{1}{\pi f} \quad . \qquad \text{(A-6)}$$

The model sketched in Fig. A-1 omits any damping mechanism. Viscous forces proportional to speed can produce decreases in resonant frequency but the change amounts to less than 20% for all degrees of this kind of damping below the critical value, when the motion becomes aperiodic. Constant friction damping produces no change in natural frequency (Ref. 5).

Another influence of more importance than damping stems from pulse shape. The relation (A-6) between the intersection at $T = T_o = 1/\pi f$ of the two asymptotes A_o and V_o and Eq. (A-5) between the magnitudes of the equivalent short impulse and the average long lasting force hold for a step in acceleration. When the acceleration pulse is a jump followed by

a steady ramp downward as illustrated in Fig. A-2, the equivalent average acceleration is one-half the value it has in Eq. (A-5). In Eq. (A-4) consider:

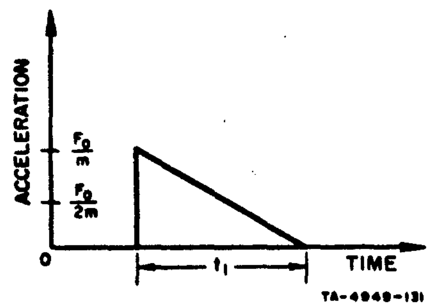

FIG. A-2

The particular integral becomes:

$$F(\tau) = F_o\left(1 - \frac{\tau}{t_1}\right) \text{ for } 0 \le \tau \le t_1$$

and

$$F(\tau) = 0 \text{ otherwise}$$

$$y_2 - \ell_o' = \frac{1}{m\omega}\int_o^t F_o\left(1 - \frac{\tau}{t_1}\right) \sin\omega(t - \tau)d\tau$$

$$= \frac{F_o}{m\omega^2}\cos\omega(t - \tau)\Big|_o^t - \frac{F_o}{m\omega^3 t_1}\int_o^t \tau\omega\sin\omega(t - \tau)d\omega\tau$$

$$= \frac{F_o}{m\omega^2}(1 - \cos\omega t) + \frac{F_o}{m\omega^3 t_1}\left\{\int_o^{x=\omega t} x[\sin x\cos\omega t - \sin\omega t\cos x]dx\right\}$$

$$= \frac{F_o}{m\omega^2}(1 - \cos\omega t) + \frac{F_o}{m\omega^3 t_1}\Big\{\cos\omega t[\sin x - x\cos x]_o^{\omega t} - \sin\omega t$$

$$\times [\cos x + x\sin x]_o^{\omega t}\Big\}$$

$$y_2 - \ell_o' = \frac{F_o}{m\omega^2}(1 - \cos \omega t) + \frac{F_o}{m\omega^3 t_1}\{\cos \omega t[-\omega t \cos \omega t + \omega t]$$

$$- \sin \omega t[+\omega t \sin \omega t - 1]\}$$

$$= \frac{F_o}{m\omega^2}(1 - \cos \omega t) + \frac{F_o}{m\omega^3 t_1}[\omega t(\cos \omega t - 1) + \sin \omega t]$$

$$= \left[\frac{F_o}{m\omega^2} - \frac{F_o t}{m\omega^2 t_1}\right](1 - \cos \omega t) + \frac{F_o}{m\omega^3 t_1} \sin \omega t$$

$$= \frac{F_o}{m\omega^2}\left\{\left[1 - \frac{t}{t_1}\right](1 - \cos \omega t) + \frac{\sin \omega t}{\omega t_1}\right\} \text{ for } 0 \le t \le t_1 \quad .$$

For the case $t_1 \gg t \gg 2\pi/\omega$

$$y_2 - \ell_o' \to \frac{F_o}{m\omega^2}(1 - \cos \omega t) \quad . \tag{A-7}$$

When t_1 is very large, the maxima of both strain and speed are reached before t_1 as shown below:

$$\dot{y}_2 = \frac{F_o}{m\omega^2}\left\{-\frac{1}{t_1}(1 - \cos \omega t) + \left(1 - \frac{t}{t_1}\right)\omega \sin \omega t + \frac{\cos \omega t}{t_1}\right\}$$

for $0 \le t \le t_1$

which at $t = t_1$ becomes

$$\dot{y}_2 = \frac{F_o}{m\omega^2}\left(-\frac{1}{t_1} + \frac{2 \cos \omega t}{t_1}\right)$$

and at $t = t_1$

$$y_2 - \ell_o' = \frac{F_o}{m\omega^2}\frac{\sin \omega t_1}{\omega t_1} \quad .$$

Thus by Eq. (A-7) maximum strain for the pulse of Fig. A-2 becomes $2F_o/m\omega^2$ but in this case the average force is $F_o/2$ or average acceleration on the base is $F_o/2m$ so that the long and short duration asymptotes in (a, v) space meet at

$$T_o = \frac{V_o}{A_o} = \frac{\frac{I_o}{m}}{\frac{F_o}{2m}} = \frac{2F_o}{\omega\frac{F_o}{2}} = \frac{2}{\pi f} \quad .$$

Kornhauser (Ref. 34) considers five basic pulse shapes: the step, the half sine wave, isoceles triangle, the decaying triangle treated above, and the rising triangle. The intersections of asymptotes for all meet in the range

$$\frac{1}{4f} \le T_o \le \frac{2}{\pi f}$$

where f is the oscillator natural frequency. Or

$$\frac{1}{4T_o} \le f \le \frac{2}{\pi T_o} \quad .$$

Appendix B

SENSITIVITY DIAGRAM FOR TWO-DEGREE OF FREEDOM COUPLED SYSTEM

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SENSITIVITY DIAGRAM FOR TWO-DEGREE OF FREEDOM COUPLED SYSTEM

Maximum distortions in the springs of a coupled two degree of freedom oscillator under a specified forcing function on the base of the compound system will be derived by a Laplace transform technique.

The coupled system is sketched in Fig. 15-c. For simplicity damping will be neglected.

If u is distortion in the primary spring and v distortion in the secondary; k_1 and k_2 are spring constants; and m_1 and m_2 masses; then the equations of motion are

$$m_1\ddot{u} + k_1 u - k_2 v = -m_1\ddot{z}$$

$$m_2(\ddot{v} + \ddot{u}) + k_2 v = -m_2\ddot{z}$$

where z is the given displacement of the base as a function of time. The transformed equations are

$$(m_1 s^2 + k_1)U - k_2 V = -m_1 \mathcal{L}\{\ddot{z}\}$$

$$m_2 s^2 U + (m_2 s^2 + k_2)V = -m_2 \mathcal{L}\{\ddot{z}\}$$

where s is the transform variable,

$$U = \mathcal{L}\{u\}$$

and

$$V = \mathcal{L}\{v\}$$

Solving, we find

$$U = \mathcal{L}\{\ddot{z}\} \frac{-(s^2 + \omega_2^2) - \frac{k_2}{m_1}}{(s - r_1)(s - r_2)(s - r_3)(s - r_4)} = -\mathcal{L}\{\ddot{z}\} f(s)$$

$$V = \mathcal{L}\{\ddot{z}\} \frac{-\frac{k_2}{m_1}}{(s - r_1)(s - r_2)(s - r_3)(s - r_4)} = -\mathcal{L}\{\ddot{z}\} g(s)$$

where r_1, r_2, r_2 and r_4 are the four distinct roots of

$$s^4 + \left(\frac{M}{m_1}\omega_2^2 + \omega_1^2\right)s^2 + \omega_1^2\omega_2^2 = 0$$

in which

$$M = m_1 + m_2$$

$$\omega_1^2 = \frac{k_1}{m_1}$$

and

$$\omega_2^2 = \frac{k_2}{m_2} \quad .$$

Further r_1, r_2, r_3 and r_4 are pure imaginary and such that

$$r_1 = -r_2 \ ,$$

$$r_3 = -r_4 \quad \text{and} \quad r_1^2 + r_3^2 = 0 \quad .$$

Let

$$\ddot{z} = A[H(t) - H(t - T_o)]$$

where A and T_o are positive constants and H(t) is a step function of time such that

$$H(t) = 1 \quad \text{for} \quad t \geq 0$$

$$H(t) = 0 \quad \text{for} \quad t < 0 \quad .$$

Then

$$\mathcal{L}\{\ddot{z}\} = A\,\frac{1 - e^{-sT_o}}{s}$$

and

$$u(t) = \mathcal{L}^{-1}\{U\} = A\left[\mathcal{L}^{-1}\left\{e^{-sT_o} f'(s)\right\} - \mathcal{L}^{-1}\{f'(s)\}\right]$$

$$v(t) = \mathcal{L}^{-1}\{V\} = A\left[\mathcal{L}^{-1}\left\{e^{-sT_o} g'(s)\right\} - \mathcal{L}^{-1}\{g'(s)\}\right]$$

$$f'(s) = \frac{1}{s} f(s) \,, \qquad g'(s) = \frac{1}{s} g(s) \quad .$$

If we let $r_1 = i\bar{r}_1$, $r_2 = i\bar{r}_2$, $r_3 = i\bar{r}_3$ and $r_4 = i\bar{r}_4$ where $\bar{r}_1$, $\bar{r}_2$, $\bar{r}_3$ and $\bar{r}_4$ are real, then

$$u(t) = \frac{A}{\omega_2^2}\left[H(\omega_2(t - T_o))F(\omega_2(t - T_o)) - F(\omega_2 t)\right]$$

where

$$F(\omega_2 t) = \left[\frac{M}{M_1}\,\frac{1}{\rho_1^2\rho_2^2} - \frac{-\rho_1^2 + \frac{M}{m_1}}{\rho_1^2(\rho_3^2 - \rho_1^2)} \cos \rho_1\omega_2 t \right.$$

$$\left. + \frac{-\rho_3^2 + \frac{M}{m_1}}{\rho_3^2(\rho_3^2 - \rho_1^2)} \cos \rho_3\omega_2 t\right]$$

$$\rho_1 = \frac{\bar{r}_1}{\omega_2}$$

$$\rho_3 = \frac{\bar{r}_3}{\omega_2}$$

and

$$v(t) = \frac{A}{\omega_2^2}\left(\frac{\omega_1}{\omega_2}\right)^2 [H(\omega_2(t - T_o))G(\omega_2(t - T_o)) - G(\omega_2 t)]$$

where

$$G(\omega_2 t) = \left[\frac{1}{\rho_1^2\rho_3^2} - \frac{\cos\ \rho_1\omega_2 t}{\rho_1^2(\rho_3^2 - \rho_1^2)} + \frac{\cos\ \rho_3\omega_2 t}{\rho_3^2(\rho_3^2 - \rho_1^2)}\right] .$$

In the foregoing terminology AT_o is the speed change on the base of the compound system. Let us measure $AT_o = v_b$ in two units. When discussing $u(t)$, v_b will be divided by the speed change $\mathring{u}_o = \omega_1 x_o$ which, applied to the base of the primary system alone, will produce maximum tolerable distortion x_o in that system; u will be measured in units of x_o. Thus

$$\frac{u(t)}{x_o} = \frac{v_b}{\mathring{u}_o}\frac{1}{\omega_2 T_o}\left(\frac{\omega_1}{\omega_2}\right)[H(\omega_2(t - T_o))F(\omega_2(t - T_o)) - F(\omega_2 t)]$$

$$= \frac{v_b}{\mathring{u}_o}\frac{1}{\omega_2 T_o}\left(\frac{\omega_1}{\omega_2}\right) F'(\omega_2 t) .$$

If F_M' is the highest value of $|F'(\omega_2 t)|$ for $t \geq 0$, then the least value of $v_b/\mathring{u}_o$ required to produce intolerable strain in the primary system becomes

$$\frac{v_b}{\mathring{u}_o} = (\omega_1 T_o)\left(\frac{\omega_2}{\omega_1}\right)^2 \frac{1}{F_M'} . \qquad \text{(B-1)}$$

Similarly when discussing $v(t)$, let $\dot{v}_o = \omega_2 y_o$ where y_o = maximum tolerable distortion in the secondary; and G'_M = the highest value of

$$G'(\omega_2 t) = [H(\omega_2(t - T_o))G(\omega_2(t - T_o)) - G(\omega_2 t)] \quad ;$$

then the least value of $v_b/\dot{v}_o$ to produce intolerable strain in the secondary becomes

$$\frac{v_b}{\dot{v}_o} = (\omega_2 T_o)\left(\frac{\omega_2}{\omega_1}\right)^2 \frac{1}{G'_M} \quad . \tag{B-2}$$

Figures 2 and 5 show

$$\frac{v_b}{\dot{u}_o \omega_1 T_o} \text{ versus } \frac{v_b}{\dot{u}_o}$$

and Figs. 3 and 6

$$\frac{v_b}{\dot{v}_o \omega_2 T_o} \text{ versus } \frac{v_b}{\dot{v}_o}$$

computed from Eqs. (B-1) and (B-2).

Appendix C

HEURISTIC EXTRAPOLATION OF MAXIMUM GROUND-TRANSMITTED MOTION

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A heuristic argument for accepting the possibility of ground-transmitted wave intensities forecast by the extrapolation toward ground zero of actual observations in the outrunning zone at Nevada, viz., intensities given by Eq. 4, Chapter V, Section D, can be given as follows:

Consider a geological environment containing a fast layer at a certain depth which produces outrunning at a range R_1 and time T_1 when 1 kt nuclear device is exploded on the surface. Over the area $R < R_1$ momentum I_1 is delivered to the fast layer without appreciable compensating loss from the layer. After outrunning begins at $R = R_1$ this momentum is then radiated into the surface in such a way that the same total amount of momentum is left behind at each radius. The total momentum per unit area on the surface above then falls inversely with the radius. Because of the mechanism of transfer from the fast layer, the rate of delivery to the surface at each radius $R > R_1$ decreases as $1/R$ also. So far the result is a peak surface speed that declines as $1/R^2$ or perhaps $1/(R - R_1)^2$ and a period of motion on the surface that increases as $R - R_1$, both properties in substantial agreement with observations.

Now, if the speed of the conducting layer can be changed without affecting the rate at which momentum is transferred from a given point in the layer to a given point on the surface above in the outrunning region, it is plausible that the intensity of the momentum transmitted to the surface point be proportional to the momentum I_1 and inversely proportional to the time T_1 required to load the layer. Thus, peak speed in the ground-transmitted motion:

$$V_M \propto \frac{I_1}{T_1(R - R_1)^2} \quad . \qquad \text{(C-1)}$$

Obviously I_1 and T_1 change in the same direction with layer speed and using empirical relations from Brode (Ref. 39) the dependence of I_1 and of T_1 upon peak overpressure can be shown to be similar when peak overpressure is 1000 psi or less. A more sophisticated analysis would probably consider a second term in the proportionality (C-1), i.e.,

$$V_M \propto \frac{I_1}{T_1(R - R_1)^2} + A$$

where A accounts for impulse put into the ground in the region of the fireball.

Brode's curves in the region $10 < P < 10^3$ psi where P = peak overpressure can be approximated by:

If I = impulse density delivered to radius R by airblast

T = time shock front reaches R then

$$\begin{aligned} R &= BP^{-0.39} \\ I &= CP^{1/2} \\ T &= DP^{-0.60} \end{aligned}$$

where B, C and D are constants depending only on airblast parameters. Hence

$$I_1 \cong 2\pi \int_0^{R_1} RdR = 2\pi A^2 B \int_{R_1}^{0} P^{0.5} P^{-0.39} (0.39) P^{-1.39} dP$$

$$\cong \pi A^2 B (0.78) \left. \frac{P^{-0.28}}{1.28} \right|_{\infty}^{P_1} = KP_1^{-0.28}$$

where the pressure at zero radius has been made infinite. Thus

$$\frac{I_1}{T_1} \propto P_1^{0.32}$$

which slowly increases as speed in the fast layer increases but, presumably, increase beyond a certain limit is prevented by the growing importance of A, which is essentially independent of layer conduction speed.

In the foregoing we have tried to make plausible the assumptions (1) that V_M due to ground-transmitted motion depends on the same power of radius from ground zero in all environments and (2) the factor of proportionality (represented above as I_1/T_1) may be only weakly dependent on the peak blast overpressure at point of first outrunning. Thus, if a seismic speed could be increased and earlier outrunning achieved, it is plausible to compute ground-transmitted wave strength V_M in the modified environment by extrapolation from the original.

Appendix D

IMPEDANCE AND DAMPING

Appendix D

IMPEDANCE AND DAMPING

Concepts of mechanical impedance apply only to steady-state sinusoidal vibrations, and in such a case when the exciting frequency is near resonance, the magnitude of damping dominates the behavior. For this reason the oscillator discussed in Appendix A is inappropriate and the physical model must include a mechanism for the conversion of kinetic energy into heat.

In engineering theory there are two simple models meeting this requirement: the series and parallel viscous systems, shown in Figs. D-1, and D-2, respectively.

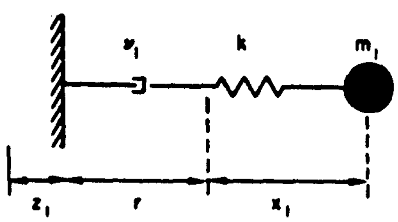

FIG. D-1

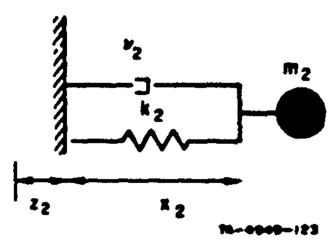

FIG. D-2

In both these models the letter m_i designates the mass concentration (the spring and dashpot are massless); k_i, the spring constant; ν_i, the coefficient of viscosity. The base is located a distance z_i from a fixed origin and is subject to sinusoidal variation; the length of the spring is x_i and length of the dashpot, where different from x_i, is r. In the series model the force F in the spring is the same as that in the dashpot, hence

$$F = -k_1(x_1 - \ell_o) \qquad \text{(D-1)}$$

$$F = -\nu_1 r \qquad \text{(D-2)}$$

The negative sign is used because both forces oppose an increase in the length of the element. Quantity ℓ_o is the unstrained length of the spring. The absolute displacment of m_1 is $z_1 + r + x_1$ so that

$$F = m_1(\ddot{z} + r + \ddot{x}_1) \quad . \tag{D-3}$$

Differentiating (B-1) and (B-2), substituting in (B-3)

$$m_1\ddot{F} + \frac{k_1 m_1}{\nu_1}\dot{F} + k_1 F = k_1 m_1 \ddot{z} \tag{D-4}$$

In Fig. D-2 the total force on m_2 is the sum $-\nu_2\dot{x}_2 - k_2(x_2 - \ell_o)$ of contributions from the dashpot and from the spring. The absolute displacement of m_2 is $z_2 + x_2$. Hence

$$m_2\ddot{x}_2 + \nu_2\dot{x}_2 + k_2(x_2 - \ell_o) = -m_2\ddot{z}_2 \quad . \tag{D-5}$$

Equations (D-4) and (D-5) are formally similar. The general solutions of the homogeneous equations are sums of exponentials, viz.:

$$F = A_1 e^{P_1 t} + B_1 e^{P_2 t}$$

$$x_2 - \ell_o = A_2 e^{q_1 t} + B_2 e^{q_2 t}$$

where

$$P_1 = \frac{k_1}{2\nu_1} + \left[\frac{k_1^2}{4\nu_1^2} - \frac{k_1}{m_1}\right]^{1/2} \tag{D-6}$$

$$P_2 = \frac{k_1}{2\nu_1} - \left[\frac{k_1^2}{4\nu_1^2} - \frac{k_1}{m_1}\right]^{1/2} \tag{D-7}$$

and

$$q_1 = \frac{\nu_2}{2m_2} + \left[\frac{\nu_2^2}{4m_2^2} - \frac{k_2}{m_2}\right]^{1/2} \qquad \text{(D-8)}$$

$$q_2 = -\frac{\nu_2}{2m_2} - \left[\frac{\nu_2^2}{4m_2^2} - \frac{k_2}{m_2}\right]^{1/2} . \qquad \text{(D-9)}$$

Since $z = z_o e^{j\omega t}$, the particular solutions can be written

$$F = F_o e^{j(\omega t+\alpha_1)}$$

and

$$x - \ell_o = x_o e^{j(\omega t+\alpha_2)} \qquad \text{[D-9(A)]}$$

where

$$\tan \alpha_1 = \frac{m_1\omega}{\nu_1} \frac{1}{\frac{m1}{k_1}\omega^2 - 1}$$

$$\tan \alpha_2 = \frac{\nu_2\omega}{k_2} \frac{1}{\omega^2 \frac{m_2}{k_2} - 1}$$

and

$$F_o = \frac{-k_1 m_1 z_o \omega^2}{\left[\left(k_1 - \omega^2 m_1\right)^2 + \left(\frac{k_1 m_1 \omega}{\nu_1}\right)^2\right]^{1/2}} \qquad \text{(D-10)}$$

$$x_o = \frac{z_o \omega^2}{\left[\frac{\nu_2^2 \omega^2}{m_2^2} + \left(\frac{k_2}{m_2} - \omega^2\right)^2\right]^{1/2}} \quad \text{(D-11)}$$

The complete solutions of (D-4) and (D-5) then are

$$F = A_1 e^{p_1 t} + B_1 e^{p_2 t} + F_o e^{j(\omega t + \alpha_1)} \quad \text{(D-12)}$$

$$x = A_2 e^{q_1 t} + B_2 e^{q_2 t} + x_o e^{j(\omega t + \alpha_2)} \quad \text{(D-13)}$$

where A_1, A_2, B_1, and B_2 are constant determined by initial conditions. However, the influence of the starting circumstances can quickly disappear because of the real components of p_1, p_2, q_1, and q_2. Equations (D-6) and (D-7) show that if $k_1/2\nu > k_1/m_1$ then p_1 and p_2 are real and both are negative. If $k_1/2\nu \geq k_1/m_1$, then p_1 and p_2 may have imaginary components but their real components are negative. Similarly, it is clear from equating (D-8) and (D-9) that q_1 and q_2 always have negative real components. Thus after a time

$$F \approx F_o e^{j(\omega t + \alpha_1)} = k_1(x_1 - \ell_o) \quad \text{(D-14)}$$

and

$$x_2 - \ell_o \approx x_o e^{j(\omega t + \alpha_2)} \quad . \quad \text{(D-15)}$$

In the presence of significant damping then motions of both masses are thus sinusoidal at the driving frequency ω. Since α_1 and α_2 are both real, the magnitudes of these responses are determined by F_o and x_o as well as by z_o, the amplitude of the driving oscillation.

Equations (D-6) and (D-7) show that in the in-line model of Fig. D-1 free oscillations are not possible when

$$\nu_1 \le \sqrt{\frac{m_1 k_1}{2}}$$

and in the model of Fig. D-2 Eqs. (D-8) and (D-9) show the same when

$$\nu_2 \ge 2\sqrt{m_2 k_2} \quad .$$

These are the values of "critical damping." The system of Fig. D-1 is underdamped when

$$\nu_1 > \sqrt{\frac{m_1 k_1}{2}}$$

and that of Fig. D-2, when

$$\nu_2 < 2\sqrt{m_2 k_2} \quad .$$

In the underdamped, critically damped and overdamped conditions the influence of starting circumstances will eventually disappear because of the negative real components of p_1, p_2, q_1, and q_2. In the first model above critical damping this disappearance proceeds more quickly the lower ν_1; the overdamped system on the contrary erases the effect of initial conditions more slowly the smaller ν_1, as can be seen by expanding the term under the radical in Eqs. (D-6) and (D-7). In the second model below critical damping rising ν_2 implies more rapid damping of original oscillations; but above critical damping the opposite is true.

Impedance J at a point in a mechanical system is the ratio of the force acting in a certain direction to the speed in the same direction. Thus at the wall

$$J_1 = \frac{F}{\dot{z}} \tag{D-16}$$

$$J_2 = \frac{k_2(x_2 - \ell_o) + \nu_2\dot{x}_2}{\dot{z}} \tag{D-17}$$

Assuming either that the influence of the starting conditions have been erased by dissipation or that motion was begun in such a way that $A_1 = B_1 = 0$, we can substitute Eq. (D-14) and the derivative of [B-9(A)] into (B-16):

$$J_1 = \frac{F_o e^{j(\omega t+\alpha_1)}}{j\omega z_o e^{j\omega t}} = \frac{k_1 m_1 e^{j(\alpha_1+\pi/2)}}{\left[\left(\frac{k_1}{\omega} - \omega m_1\right)^2 + \left(\frac{k_1 m_1}{\nu_1}\right)^2\right]^{1/2}} \tag{D-18}$$

Regardless of the degree of damping $|J_1|$ reaches a peak at the resonance frequency $\omega^2 = k_1/m_1$. The phase φ_1 of J_1 is simply given by:

$$\tan\phi_1 = \frac{-1}{\tan\alpha_1} = \nu_1\left(\frac{1}{m_1\omega} - \frac{\omega}{k_1}\right) \tag{D-19}$$

At $\omega = 0$, ϕ is $\pi/2$ (or $3\pi/2$). As ω increases ϕ_1 decreases until at resonance $\phi_1 = 0$ (or π) after which $\phi_1 \to -\pi/2$ (or $\pi/2$).

Substituting (D-15) and [D-9(A)] into (D-17)

$$J_2 = \frac{k_2 x_o e^{j(\omega t+\alpha_2)} + \nu_2 x_o j\omega^{j(\omega t+\alpha_2)}}{z_o j\omega e^{j\omega t}}$$

which after substitution for x_o and α_2 and considerable manipulation becomes

$$J_2 = \frac{\left\{-\nu_2 m_2\omega - j\left[\frac{k_2}{\omega}\left(\frac{k_2}{\omega} - m_2\omega\right) + \nu_2^2\right]\right\} m_2\omega}{\nu_2^2 + \left(\frac{k_2}{\omega} - m_2\omega\right)^2}$$

J_2 reached a maximum near $\omega^2 = k_2/m_2$ but the exact location can be shifted by the value of ν_2. When $\nu_2 \to 0$ and $\nu_1 \to \infty$ then $|J_1| \to |J_2|$. As long as ν_1 is bounded, J_1 is finite at resonance; J_2 is bounded provided $\nu_2 > 0$.

Phase angle ϕ_2 of J_2 is found from the formula

$$\tan \phi_2 = \frac{\frac{k_2}{\omega}\left(\frac{k_2}{\omega} - m_2\omega\right) + \nu_2^2}{\nu_2 m_2 \omega} = \frac{k_2}{\nu m_2 \omega^2}\left(\frac{k_2}{\omega} - m_2\omega\right) + \frac{\nu_2}{m_2\omega} \quad .$$

Thus when $\omega = 0$, $\phi_2 = \pi/2$ (or $3\pi/2$), and as ω rises, ϕ_2 falls toward zero (or π) which it reaches when and if

$$\omega^2 = \frac{k_2}{m_2} \frac{1}{1 - \frac{\nu_2^2}{k_2 m_2}} \quad . \qquad \text{(D-20)}$$

When $\nu_2 = 2(k_2 m_2)1/2$, damping is said to be critical because free oscillations become impossible. We see from the equation above that if damping is half critical or greater, ϕ_2 never reaches zero (or π) as $\omega \to \infty$. If $\nu_2^2 < k_2 m_2$ then ω increases beyond the value given by (20) ϕ_2 decreases but eventually turns around and goes to zero (or π) as $\omega \to \infty$.

Differing phase behavior as $\omega \to \infty$ distinguishes the two models with viscous loss mechanisms. Which model applies more closely to the human body or to any part of the human body is not clear. Dieckmann (Ref. 10) says that, except for frequencies below Hz and for static loading, the in-line model(Fig. D-1) is "adequate and accurate"; and his measurements of phase change with frequency show a full 180° shift through the range 1 to 100 Hz for both sitting and standing men vibrated vertically. Von Bekesy (Ref. 65) finds the magnitudes of the impedances for standing and sitting men to have peaks near 10 Hz instead of in the range 4 to 5 Hz as Dieckmann found, and von Bekesy reports a phase behavior more

like that outlined above for the model in Fig. D-2, viz., as ω increases from zero the phase angle passes from $\pi/2$ to 0 and slightly below zero before increasing again. He does not however show the phase as approaching zero after resonance has been passed but as actually reaching zero and going beyond to positive angles.

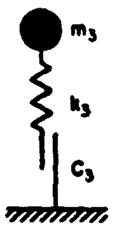

FIG. D-3

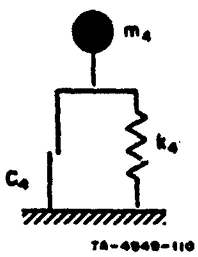

FIG. D-4

Certainly the actual bodily reaction may demand a combination of several of the simple models for its accurate representation. Coulomb or constant friction as opposed to viscous friction is also sometimes considered. As shown in Figs. D-3 and D-4 models in analogy with those of Figs. D-1 and D-2 can be studied.

In these the constant force C always opposes the motion of the elements of which it is a part. Generally, behavior of these purely coulomb models does not compare with that of actual physical systems since resonances tend to be narrow and strong and phase shifts very small over wide frequency ranges and strongly discontinuous at resonance. Coulomb elements are most useful as components of systems which are basically those of Figs. D-1 and D-2.

Appendix E

SHELTER EQUIPMENT TEST RESULTS

Table E-1

SUMMARY OF SHOCK TESTING FOR

DESCRIPTION OF EQUIPMENT	SIZE	MANUFACTURER/MANUFACTURER'S DESIGNATION	DESCRIPTION OF T
1. Annunciator	3' x 4' x 1 2' (?)	Panalarm Division of Panellet, Inc., Skokie, Ill. Model 5127 (125 v DC)	LWHI, 11" drop, side to side, 5-1/2" drop, front to 1° drop, top to bottom
2. Annunciator	As above	Panalarm Division of Panellet, Inc., Skokie, Ill. Model 5136 (115 v AC)	LWHI, (1-1/2" drop, side to s 9" drop, front to back 22" drop, top to bottom
3. Auto Roll Unit with Electric Motor (Air Conditioning Equipment)	110 v 1 ϕ ~ 3' x 6' x 2'	Electro-Air Cleaner, Inc.	MWHI, 1' drop, 2' drop } 100 g max, ve
4. Batteries (2) Ni-Cd	15" x 15" x 6"	Nife, Inc., Copiague, N.Y. KBI-25	LWHI, 12" drop, Y-axis, 3" drop, X-axis, 3-1/2" drop, Z-axis }
5. Centrifugal Fan	2700 lb	Bayley Blower Co., Milwaukee, Wis. Model BC-TH	Rolling table for horiz. test reported Drop for vert., height not re 7 g max (?), both axes
	1375 lb	Bayley Blower Co., Milwaukee, Wis. Model BC-UB	Direct drive mechanical vibra 7 g max, 3 axes Frequencies not reported
	1175 lb	Bayley Blower Co., Milwaukee, Wis. Model BC-TH	Same as above
	850 lb	Bayley Blower Co., Milwaukee, Wis. Model BC-UB	Same as above
	250 lb	Bayley Blower Co., Milwaukee, Wis. Model BC-UB	Same as above
6. Centrifugal Pump and Motor	total weight 1650 lb overall length ~ 5'	Allis Chalmers, Milwaukee, Wis. Model 211-494-502 Size 5 x 4 Type SJ	MWHI, 1.25' and 2.25' drops,
	60 hp, 1750 rpm 3 ϕ, 60 Hz	Allis Chalmers, Norwood, O. NEMA Des. B Type AP	
7. Compressor and Motor	2-1/2" x 2-1/2" 3/4 hp	Quincy Compressor Co., Quincy, Ill. A4-377955	LWHI, 20 g peak, all axes
	1750 rpm 115/230 v 1 ϕ, 60 Hz	Century Electric Co., St. Louis, Mo. CS-68-FHK3-3FA	
8. Converter (Heater)	150 psi, 375°F ~ 6' x 1' dia.	Taco Heaters Inc., Cranston, R.I. 14212SR-B	MWHI, 19 to 23 g peak* throug low pass filter, 3 axes
9. Distribution Panel (with Circuit Breakers)	10' x 4' x 1' (?)	General Electric Co., Schenectady, N.Y. Panel "A"	MWHI, 3" drop, vert., 20 g ma 3" drop, side, 20 g max 6" drop, back, 22.5 g m
10. Distribution Panel (with Circuit Breakers)	5' x 4' x 1' (?)	General Electric Co., Schenectady, N.Y. Panel LP-ULB	MWHI, 2-3/4" drop, vert. 21.5 4" drop, side, 20 g max 6" drop, back, 22 g max
11. Light Fixtures: Lightalarm Battery Charging Unit Recessed Downlight Vaportite Ceiling Unit Vaportite Bracket Turnlox Reflector Unit Compact Directionals Compact Directionals 40 W Rapid Start Weatherproof Bulletin Luminaire Skylouver		General Electric Co., Schenectady, N.Y. WP402NCVA 4-601A McPhilben N-43-45 VT McPhilben N-43-40 VT McPhilben G-7642 Benjamin 50W-6M McPhilben 50E-6M McPhilben A1 Benjamin K-44981 Simes CFR Guardian 2047 LPF-TS Electrolight	MWHI, vert. or Z—4" drop, 22 fixture LWHI, horiz. or X and Y—vari 4" to 13", 30+ g max
12. Motor Control Center	44" x 90-3/8" x 20-1/4" 1900 lb 440 v operating 110 v control	Westinghouse Electric Co., St. Louis, Mo. Class 11-350	VDMI, Free fall into sand wit as stoppers, 7-1/2" fall, 50 g max, vert. 20 g max, horiz. Frequency search

LWHI = light weight high impact test machine MWHI = medium weight high impact test machine VDMI = variable d

A.

Table E-1

OCK TESTING FOR EOC, ALBANY, N.Y.

DESCRIPTION OF TEST	RESULT	COMMENT
" drop, side to side, 25 g max -1/2" drop, front to back, 19 g max " drop, top to bottom, 80 g max*	No damage or operational impairment	Test adequate for airslap and probably also for all components of ground motion since equipment is small
-1/2" drop, side to side, 19 g max " drop, front to back, 28 g max " drop, top to bottom, 19 g max	Panel light operation unimpaired Mechanical damage: 3 indicators fell off	As above Mechanical damage not serious
drop drop } 100 g max, vert. only (?)	Base of filter drive motor broke; otherwise no damage	Because of size, equipment may have low modal frequencies; test probably inadequate for strong ground-transmitted motion
" drop, Y-axis " drop, X-axis -1/2" drop, Z-axis } 20+ g max	No damage	Why do 3" drops produce same acceleration of same equipment as 12" drops? Doubt peak g measurement applied to whole equipment
table for horiz. test; speed not ed vert., height not reported (?), both axes	No damage or operational impairment	From data given adequacy at test not conclusive
rive mechanical vibration machine 3 axes ies not reported	Same as above	Same as above
above	Same as above	Same as above
above	Same as above	Same as above
above	Same as above	Same as above
25' and 2.25' drops, vert. only (?)	No operational impairment Weld cracked open on mounting foot	Vert. modes probably adequately tested for airslap
g peak, all axes	No damage	Adequate for airslap, probably also adequate for all likely motion of this small equipment
to 23 g peak* through 80 Hz ss filter, 3 axes	Mounting bolts severely bent during blows parallel longitudinal axis No operational impairment	Probably adequate test for airslap. Behavior at low frequencies not established.
drop, vert., 20 g max† drop, side, 20 g max† drop, back, 22.5 g max†	No damage or operational impairment	This large equipment should be tested for resistance to ground-transmitted motion also (Equipment had slight modifications for shock service)
3/4" drop, vert. 21.5 g max† drop, side, 20 g max† drop, back, 22 g max†	Same as above	Same as above
t. or Z—4" drop, 22 g max on iz. or X and Y—various drops 13", 30+ g max	No damage except to Bulletin Luminaire, which must be shock mounted Heavy duty bulb required in 40 W Rapid Start	Probably adequate test for this equipment except for possible swing modes in suspended ceiling fixtures
e fall into sand with maple blocks pers, 7-1/2" fall, x, vert. x, horiz.	Angle clip holding control device panel broke; corrected by redesign Tendencies for two breakers to trip on vert. blow; corrected by choosing larger breaker Tendency for momentary closure of breakers on transverse blows; corrected with anti-shock latch Otherwise no damage	Device as modified adequately tested for airslap
y search	Equipment has frequency at 9 Hz; changed to 25 Hz by redesign	

VDMI = variable duration, medium impact test machine

B

Table E-1 (Concl

DESCRIPTION OF EQUIPMENT	SIZE	MANUFACTURER MANUFACTURER'S DESIGNATION	DESCRIPTION C
13. Motors (Electric) 1800 rpm	10 hp 15 hp 25 hp 75 hp	Not reported 5K4256A21 5K4284A22 5K4324A21 5K4405A2	All dropped 6" in normal plate: 189 g max through 25 Hz 1100 g max through 2000 For horiz. tolerance moto nels, which were wheele on wall: 7 g max throu filter both horiz. dire speed change
14. Motor-Generator (Diesel) 4 cycle 6 cylinder 9" x 10-1/2" bore and stroke 900 rpm, 60	40,000 lb 580 Bhp 500 KVA 400 KW	Chicago Pneumatic Tool Co., N.Y., N.Y. 69-CPHS	RR car impact, 12 MPH, 7 and transverse 23" free fall at motor en 19" free fall at generato Duration of deceleration
15. Motor Sentinel	4-3/4" x 2-3/4" x 2-3/4" 1 lb	Westinghouse Electric Corp., Beaver, Pa.	VDMI, free fall into sand as stoppers, 13" free f axes
16. Panelboards§ MS-E2-L1 MS-E2-L2 Control Supply Tel. Cab. Tel. Cab. Tel. Cab.	19-1/4" x 20" x 7-1/2" Same as above Same as above 36" x 18" x 4" 6" x 6" x 4" 28" x 10" x 4"	Westinghouse Electric Co., St. Louis, Mo.	VDMI, free fall into sand as arresters , 50 g pea
17. Panelboards§	19-1/4" x 20" x 7-1/2" 24-3/4" x 20" x 7-1/2" 24-3/4" x 20" x 7-1/2" 38-1/4" x 20" x 7-1/2" 44" x 20" x 7-1/2" Same as above	Westinghouse Electric Co., St. Louis, Mo. CCP PH-PE2 CCE-2 PH-PA CCA—Section #1 CCA—Section #2	VDMI, free fall into sand as arresters, 25 g peak
18. Pump and Induction Motor	5 hp 1740 rpm 3 φ, 60 Hz 150 gpm 42 ft TDH	Continental Electric Co., Rockford, Ill. WV 215C Food Mach. & Chem. Corp., Chicago, Ill. LMC-4	MWHI, drop height not rep 6 to 8 g max† through 8 filter, 3 axes
19. Pump and Motor (Submersible)	2 hp 440 v 3 φ, 60 Hz 20 gpm 4"	Franklin Electric Co., Inc. Model 3P1078B4D Peerless Pump Division Food Mach. & Chem. Corp., Indianapolis, Ind. Model 420D200	LWHI, drop not reported 20 to 23 g max† througt filter, 3 axes
20. Pump and Motor	100 hp 3 φ, 60 Hz 440 v 3450 rpm	Buffalo Pumps, N. Tonawanna, N.Y. Reliance Electric & Engineering Co., Cleveland, O.	MWHI, 2.25' drop, } 100 g 1.25' drop,
21. Switchgear Assembly Hand-Operated Breakers Low-Voltage	3 sections x 27" wide 4 sections x 20" wide 3 sections x 26" wide 4 sections x 20" wide	General Electric Co., Philadelphia, Pa. AKD-5 AK-2A-50 } breakers AK-2A-25 AKD AK-2-50 } breakers AK-2-25	MWHI, 4" drop, 40 g max 20 g max MWHI, 10" drop, 100 g ma 50 g ma
22. Transformer 3 φ, 60 ~	225 KVA	General Electric Co., Schenectady, N.Y.	MWHI, 5" drops, 25 to 27
23. Waterchilling System	12,300 lb (dry)	Borg-Warner, York, Pa. HT 24	Free fall onto springs,△ 6-7/8" fall, 13/16" sp 7 g max, vert. Pendulum swing into spri 63" to 70" swing, 9/16 defl., 7 g max both ho
24. Waterchilling System	30,500 lb (dry)	Borg-Warner, York, Pa. MT 25	Free fall onto springs,△ 6-5/8", 17/16" spring Pendulum swings into spr 66-1/2 to 70-3/8 swi defl., 7 g max both ho

* Measured at anvil; other readings on equipment.

† Location of accelerometer not reported.

§ Panels were attached to shock pl

△ Vibration Mountings, Inc., Type

A.

ıble E-1 (Concluded)

DESCRIPTION OF TEST	RESULT	COMMENT
pped 6" in normal position onto steel : max through 25 Hz low pass filter g max through 2000 Hz low pass filter iz. tolerance motors mounted on chan- which were wheeled into shock mounts ll: 7 g max through 25 Hz low pass r both horiz. directions, 20 in./sec change	No damage nor operational impairment	Horiz. tests barely adequate for airslap but equipment probably withstands all likely ground motion
impact, 12 MPH, 7 g max, longitudinal ransverse e fall at motor end only, 6.6 g max e fall at generator end only, 7 g max ı of deceleration 1 to 2 sec	Overheating due to fouled lube line (impact loosen scale and weld splatter); corrected by stainless steel line and cleaning Gage glass loosened—not disabling No observable permanent bending or lessening of clearances	Good test for airslap at ~ 25 psi, probably not severe enough for 50 psi or for strong ground-transmitted motion in all environments
ree fall into sand with maple blocks oppers, 13" free fall, 50 g max, all	No damage	Adequate for airslap, probably also adequate for all likely motion of this small equipment
ree fall into sand with maple blocks resters , 50 g peak, all axes	No damage; no false opening or closing	Tests probably adequate for both airslap and strong ground-transmitted motion due to small size of equipment. Should also be tested in racks.
ree fall into sand with maple blocks resters, 25 g peak, all axes	No damage; no false opening or closing	Same as above
rop height not reported 3 g max† through 80 Hz low pass r, 3 axes	No damage or operational impairment	Reaction to frequencies below 20 Hz may not have been adequately tested (Shape of equipment: cylindrical ~ 5" x 8" dia.) Test not quite adequate for airslap above 50 Hz
rop not reported 23 g max† through 80 Hz low pass r, 3 axes	No damage or operational impairment	Probably adequate for this equipment (Pump size ~ 36" x 4" dia.)
.25" drop, .25" drop, } 100 g max, vert. only (?)	Performance unaffected Slight increase in mechanical unbalance in pump (?)	Because of rotational symmetry of equipment, vert. testing probably enough Adequacy of test below 20 to 30 Hz doubtful (Equipment probably has frequencies below 30 Hz)
" drop, 40 g max vert. 20 g max horiz. 0" drop, 100 g max, vert. 50 g max, horiz.	No damage, closed breakers may open above 15 g	Probably adequate for airslap only. Cabinetry may have low frequency responses not explored
' drops, 25 to 27 g peak	No damage	Probably adequate test for this equipment
ll onto springs,△ ' fall, 13/16" spring defl., ax, vert. s swing into springs,△ › 70" swing, 9/16" to 17/16" spring . 7 g max both horiz. axes	No damage nor operational impairment	Not enough data to determine adequacy for this particular equipment, but test motion does not properly simulate either airslap or ground-transmitted motion; fragility level not established; this large equipment probably has frequencies especially susceptible to strong ground-transmitted motion
ll onto springs,△ ', 17/16" spring defl., 7 g max, vert. ı swings into springs△ 2 to 70-3/8 swing, 9/16" spring 7 g max both horiz. axes	Same as above	Same as above

.tached to shock platform through cupmounts made by Barry Controls, Inc., Cat. Nos. 1010, 1015, 1035.

ıtings, Inc., Type SYA-20, 24,700 lb/in., 4 used with HT 24, 8 used with MT 25.

B.

Table E-2

NOMINAL FRAGILITY LEVELS USED FOR EOC, ALBANY, N.Y.

DESCRIPTION OF EQUIPMENT	SIZE	MANUFACTURER/ MANUFACTURER'S DESIG.	STATED FRAGILITY LEVEL
1. Blower (garbage room)	92 lb	Brunner Div. Dunham Bush, Inc. WJ-45	10g, 3 axes (?)
2. Cabinet, hydraulic jack	23" x 16" x 7" 95 lb	Not reported	3g.
3. Cabinet, motor controller	108 lb ~ 28" x 13" x 14"	Mosler Safe Co., Hamilton, O. (?)	3g
4. Compressor (refrigerated garbage)	156 lb	Brunner Div. Dunham Bush, Inc. WC-50-FC	10g, 3 axes (?)
5. Dishwasher	1000 lb (inc. water)	Toledo TA-27	6g, 3 axes (?)
6. Hydraulic jack (blast door)	16 ton capacity 48" stroke 465 lb	Richard Dudgeon (?)	> 18.8g w/door closed and locked < 18.8g w/door open
7. Hydraulic pump	997 lb ~ 44" x 22" x 37"	Not reported	3g
8. Purge compressor	120 lb 11" x 11" x 18½"	York	$\geq$ 1.7g vert.
9. Radiator	2100 lb 846 rpm	Young Radiator Co. model 1450	3.5g, 3 axes (?)
10. Range (electric)	390 lb	Hotpoint model HRG 13	3.5g, 3 axes (?)
11. Refrigerator and freezer	~ 1000 lb (loaded)	Victory Metal Manu. Co. model VS-48-S model VFS-48-S	3.5g, 3 axes (?)

Table E-3

SHOCK RESISTANCE CERTIFICATION FOR CITY AND STATE EOC'S, OKLAHOMA CITY, OKLA.

DESCRIPTION OF EQUIPMENT	SIZE	MANUFACTURER/ MANUFACTURER'S DESIG.	DESCRIPTION OF TEST	RESULT	COMMENT
1. Air Filte:	7' x 7'	American Air Filter Co. Louisville, Ky. 7-70 Roll-o-matic model B	MWHI, 1.25' drop, 3" table travel 2.25' drop, 1.5" table travel	No operational impairment Slight modification of structure required	Probably an adequate test even though low frequency response not explored
2. Fan w/electric motor (centrifugal)	12000 cfm 1650 lb	American Blower Corp. Detroit model no. C12A	MWHI, 1.5', 2.5', 3.5', 4.5', and 5' drops	Required stronger mount otherwise no operational impairment	Probably an adequate test even though low frequency response not explored
3. Pump and motor (centrifugal)	717 lb 10 hp	Westinghouse WS-107A-2 model no. 3655	VDMI 13" drop into sand, 3 axes 7 wooden blocks as arrestors	No observable damage (dye-penetrant test applied)	Peak accel. during test somewhat below maximum possible in field otherwise comment same as above

MWHI = medium weight high impact test machine

VDMI = variable duration, medium impact test machine

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The study investigated the potential hazard created by ground motion from a nearby nuclear explosion to people and equipment inside a hardened personnel or civil defense command and control shelter which itself withstands both the airblast and ground shock. Data on the fragility levels of people and equipment were extrapolated from experiments in other environments using the concepts of the simple harmonic oscillator and the steady-state, one-dimensional shock wave. Results indicated that a general population can be sheltered in a 50-psi environment but that some bodily injury may occur, chiefly caused by loss of balance. Protective postures (bent knees, prone forms, etc.) will offer greater resistance to damage-inducing motions than will standing. Low impedance covering should be installed on shelter walls and floors and sharp edges eliminated

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The study investigated the potential hazard created by ground motion from a nearby nuclear explosion to people and equipment inside a hardened personnel or civil defense command and control shelter which itself withstands both the airblast and ground shock. Data on the fragility levels of people and equipment were extrapolated from experiments in other environments using the concepts of the simple harmonic oscillator and the steady-state, one-dimensional shock wave. Results indicated that a general population can be sheltered in a 50-psi environment but that some bodily injury may occur, chiefly caused by loss of balance. Protective postures (bent knees, prone forms, etc.) will offer greater resistance to damage-inducing motions than will standing. Low impedance covering should be installed on shelter walls and floors and sharp edges eliminated to minimize bodily injury to toppling forms. Small pieces of equipment appear to be able to survive up to 50 psi, but large pieces of equipment may be susceptible to damage below a frequency between 10 and 50 Hz, depending on overpressure. Hardmounting of equipment in shelters is recommended as well as wider use of drop and variable duration tests to simulate ground motion effects on equipment.

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